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Degree: Master Science

Year this Degree Granted: 2001

University of Alberta

Broadband Integrated Services of Digital Network Optimization with Stochastic Traffic Flows

by

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A thesis
submitted to the faculty of graduate studies and research in partial fulfillment of the requirements of the degree of
Master Science

Department of Electrical and Computer Engineering

Edmonton, Alberta
Fall 2001

University of Alberta

Faculty of Graduate Studies and Research

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled Broadband Integrated Services of Digital Network Optimization with Stochastic Traffic Flows submitted by Qun Zhao in partial fulfillment of the requirements for the degree of Master Science.

Abstract

Modern communication paradigms are turning to B-ISDN protocols based on MC flow approaches. One of the representative realizations is ATM technology. Optimal design of B-ISDNs is a challenging task because of diverse traffic patterns produced by multiple communication sources.

Interest in optimal design of B-ISDNs has been high. Most proposed approaches has been developed using two disciplines of mathematics: *Graph Theory* and *Mathematical Programming* (MP). There is a need to introduce a new methodology to characterize the stochastic features of traffic behaviors in B-ISDNs.

In this thesis, the main focus is to integrate the *Stochastic Programming* (SP) methodology into MC network models. Particularly, the *Wait-and-See* (WP) approach of SP is adapted into MC network analysis. Generic FA models are presented. Probability distribution paradigms are detailed. Metrics reflecting the effect of stochastic traffic demands are proposed. Simulations are conducted for three case studies with an already available nonlinear programming software package.

Broadband Integrated Services of Digital Network Optimization with Stochastic Traffic Flows

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ATM	Asynchronous Transfer Mode
B-ISDN	Broadband Integrated Service Digital Network
CA	Capacity Assignment
CFA	Capacity and Flow Assignment
DR	Difference Ratio
FA	Flow (Routing) Assignment
ISDN	Integrated Service of Digital Network
MC	Multi-Commodity
OR	Operations Research
PDF	Probability Distribution Function
SA	Sensitivity Analysis
SLPM	Stochastic Linear Programming Methodology
VC	Virtual Channel
VP	Virtual Path

Chapter 1 Introduction

B-ISDN technology is an extension of ISDN and is developed to support a large variety of services such as Video Telephony, Video Surveillance, High Volume File Transfer and High Definition Television.

A switching technique known as ATM has been adopted as the transfer mode for B-ISDN. Unlike other switching packet technology like X.25 or Frame Relay, the ATM uses fixed length packets called ATM cells. Each ATM cell has 53 bytes, with a 5-byte header for the address and a 48-byte information field. The outstanding advantage of this technology is its ability to support a high switching rate, because switching is done purely at a hardware level without the assistance from software interface. Additionally, by using fixed length cells, the prediction of the size of the buffers can be used to transmit at convenient rate.

Furthermore, fiber optic cables replacing existing twisted pair copper wires has widely been adopted to reduce the delay caused by the limited bandwidth by existing twisted pair copper wires. The requirements for error detection/correction protocols might be eliminated for the need for extensive layers of protection in the high-speed transmission media.

ATM technology is a connection-oriented service in packet switched networks. Before any communication is set up, the end-to-end connection is established which is known as a VC connection. Then, the performance of network will be improved when the connection sharing common paths are grouped into a single unit, which is known as VP.

For last several years, VP design and management have been the hottest and interesting topic during the development of ATM technology. A careful selection of VPs may result in a VP layout that provides a reliable scheme of VCs planing with low connection and switching cost. Meanwhile, such a network is able to successfully deal with unexpected

traffic loads. However, due to the traffic complexity supported by B-ISDN/ATM, the VPs planning/optimization has been proved to be a difficult task.

OR plays an important role in solving the optimization problems of mathematical models extracted from practical applications. Its general definition can be considered as the application of the approaches of science to complex problems describing the operation and management of large systems with several important factors, such as men, machines, materials and money. The distinctive approach is applied to a scientific model of the system, also incorporates some measurements like chance and risk, and then predicts and compares the final decisions on alternative planning, strategies or controls.

General programming problems concern with allocation of scarce resources, such as labor, material, machines, and capital, and, in the "best" possible manner, so that costs are minimized or profits are maximized. In using the term "best" it is implied that some choice or a set of alternative courses of actions is available for making the decision. In general, the best decision is found by solving a mathematical problem. The term "linear programming" merely defines a particular class of programming problems that meet the following conditions:

Minimize $\quad C^T x$

Subject to $\quad A x = B$

$\qquad\qquad\quad x >= 0$

Where x is the vector of variables to be solved for, A, C and B are the matrices with known coefficients. The function being minimized (or maximized) is called the 'objective function', and the equations are referred to as 'constraints'. Particularly, $x \geq 0$ is called non-negativity constraints.

Modern communication paradigms, requiring high bit rates for transferring voice, video, and data information, are turning to the B-ISDN protocols based on *Multi-Commodity* (MC) flow approaches. Traditional network optimization models are being challenged, and the stochastic characterizations of traffic arrivals in network models have been employed for network optimization problems. The conventional optimization approach

by using mean value of models is becoming invalid. The stochastic model, such as SP problem, is considered more reasonably to match and be current practical solution. In real situation, external traffic flows are typically random variables and shall be determined optimally. In later part of this thesis, SP approach is integrated into our network optimization problem. This is one of our contributions in this thesis.

In general, network optimization models typically involve two indices: operational index and capital index. The operational index is the delay T of packets or the number of packets in the system, and capital index is the cost D per unit capacity. Then, for a network optimization problem, one of these two indices may be used as the objective functions, expressed with entities called design variables. There could be several sets of constraints, one of which could be that link flows are not greater than link capacities.

In modern communication studies, the Poisson model has been considered to be an appropriate approximation for traffic patterns in virtual circuit switching such as call connection scenarios. In this thesis, the Poisson traffic model is integrated SP methodology to obtain MC network models through three case studies.

In the case studies of this thesis, a SA procedure is developed to investigate the ranges on changing both objective function coefficients (path cost) and constraint values (path capacity and external traffic) over which the values of decisions variables (in an optimal solution) will remain unchanged. Furthermore, as our discussion in SP and its considerations on the probability distribution of optimal solutions, a measurement of variance on optimal solutions, particularly on their statistical characterizations such as PDF, is designed and described in details. In general, based on the combine approaches of SA and SP, we develop a new measurement model to investigate overall characterizations of FA problem. This is also a main contribution throughout our research project on B-ISDN optimization.

In this model, a metric called the DR is developed to characterize the changing trend of the PDF under various realizations of cost, capacity and external traffic. It is used to

express the degree of similarity between two measured PDFs. Lower value of DR refers to very similar situation between two measured PDFs. Particularly, this concept can also be extended to further meaning in our FA problems. For example, lower value of DR states similar PDFs between optimal flow solution and external traffic flow. This path with lower DR is assigned more traffic flow than other paths with higher DR in same OD pair.

As the statement on the property of performance function (objective function), the performance will be increased whenever the sets of flow approach the capacity of their channels (or say, their upper boundary for these flows defined by the capacity constraints). In three simulation cases, when path traffic approaches the upper boundary of path capacity (with lowest cost), in an optimal manner, other paths are then assigned to handle more traffic even though they are with higher cost.

Furthermore, for various realizations of both α (capacity constraint and external traffic) and β (cost coefficients), SA and WS were conducted to estimate the model characterizations of sensitivity. The results on our optimization problems exhibit the relative small ratio of the variance of path capacities to the variance of path costs. This implies that the optimal path flow is sensitive to the selection of the given design for cost coefficients over path capacities.

This thesis introduces the stochastic programming methodology to deal with the probabilistic unfeasibility issues in network management. The rest of this thesis is organized as follows: In chapter 2, an overview of B-ISDN and ATM technology is presented, then SLPM and relevant background are also discussed. Moreover, Tele-traffic Modeling of B-ISDN/ATM Telecommunications is introduced and a model based on the Poisson process is developed. Chapter 3 is to describe network models and problem statements. Then, two case studies are presented. One of the SP approaches, the WS method, is integrated into the presented model analysis. Simulation results are reported. Finally, conclusions are included in chapter 4.

Chapter 2 Background Knowledge

2.1 ISDN and ATM

2.1.1 ISDN and Broad ISDN

In modern application of telecommunications, digital technology is widely adopted to transfer information through a variety of networks to the end users. One typical application is ISDN. Generally speaking, there are two forms of ISDN service [1][20].

Narrowband ISDN

Narrowband ISDN is digital service with the transport rates of 1.544Mbs (T1) or less. The following services are included in Narrowband ISDN.

Circuit Switched Voice

Circuit Switched Voice service provides digital voice service with many of the capabilities of a business Centrex over a 4-wire ISDN Digital Subscriber Line (DSL).

Circuit Switched Data

Circuit Switched Data service provides end-to-end digital service, data or video information are transferred over the public telecommunication network. By using special processing in out-of-band signaling, ISDN set up and maintain data connections.

Low Speed Packet

ISDN connections are transmitted over packaged information by D channel at 16Kbs X.25 connection.

High Speed Packet

ISDN connections are equipped with two B channels. Circuit Switched Voice, Circuit Switched Data, or High Speed Packet Service are transferred by B channels with 64 Kbs. Moreover, one or both the 64Kbs B channels are permanent virtual circuit to the packet network and able to provide high speed packet service by 64Kbs X.25 connection.

Broadband ISDN Service

Broadband ISDN service is applied to digital services with higher rates like more than 1.544 Mbs. These digital services include in the form of Frame Relay, SMDS, or ATM. The Broadband ISDN services support wide range of transfer rates such as from 25 Mbs up to the Gigabit. Broadband ISDN Service therefore makes the high quality of the digital service possible and is considered as the service of the future. Currently it is being applied to the many applications of the Information Super Highway.

The developments of latest digital facilities such as fiber optics help to significantly improve the performance of high speed broadband services and eliminated some network operations such as, X.25 with extensive overhead, error correction and flow control performed at intermittent points. Indeed, these functions have been handed to the upper layer on an end-to-end basis.

Frame Relay

Frame Relay service supports the transport of data and is a connectionless service. Particularly, each data packet routs through the network and contains address information. A variety of rates from 56 Kbs up to 25 Mbs and variable size data packet services are supported by Frame Relay.

Switched Multimegabit Digital Service (SMDS)

SMDS is a digital service that provides a high speed digital path for permanent virtual circuits. The transport speed for SMDS is usually 155 Mbs.

Asynchronous Transfer Mode (ATM)

ATM is considered by most to be the transport service of the future. In most ATM applications, the transport rates are from 155 Mbs up to 622 Mbs. The cell size for all applications is 53 bytes. The details will be included in the following sections.

2.1.2 Asynchronous Transfer Mode (ATM)

During the 80's, the Integrated Services Digital Network (ISDN) could operate at 64 kbps at a basic channel that (B-channel) or the combinations of others (D channels). This was formed as the basis of communication on the network. Later, more and more applications, such as high-speed packet communication and video communication, had been leading to the high demand for high-speed and broadband services. Thus, Broadband-ISDN was developed as conceptually an extension of ISDN, and can support integrated broadband services like high-speed-data service, videophone, video conferencing, CATV services as well as traditional ISDN services such as phone and telex. Various services are with transmission and switch speeds from 155Mbps, 622Mbps and up to 2.4Gbps. Thus, based on the previous solutions like SDH for transmission and cell relay for switching problem, ATM is developed and improved to operate and manage the complicated situations in modern network like switching signals ranging from 10s of bps to 100s of Mbps and service time distribution ranging from a few seconds to several hours[1][20].

To have the B-ISDN services mentioned above, an interface between the ATM layer and higher layers was necessary. The ATM adaptation layer provides this service. Its main purpose is to resolve any disparity between a service required by the user and services available at the ATM layer. It lies between the ATM layer and the higher layers of the B-ISDN protocol reference model.

The fixed length of ATM cell is the basic unit of information transfer in the B-ISDN ATM protocol. Each cell is comprised of 53 bytes. Among them, five of the bytes make up the header field, and the remaining 48 bytes form the user information field.

Each ATM cells are transported via virtual channels and indirectly in virtual paths. Typically, virtual channel is considered as a unidirectional pipe, and virtual path is made to contain a set of virtual channels. Finally, ATM cells from ATM layer are collected, organized and routed through physical medium, such as fiber optic cable, by the physical layer

ATM Traffic Control

Various applications such as video conferencing needs a guaranteed amount of bandwidth available for the communications channels, meanwhile the networks should be operated as efficiently as possible and cope with potential errors at any time. Thus, certain traffic control capabilities are necessary to deal with the complicated situations in ATM network. Regularly, The network should have the following traffic control capabilities:

- Network Resource Management
- Connection Admission Control
- Usage Parameter Control and Network Parameter Control
- Priority Control
- Traffic Shaping
- Congestion Control

Traffic Control Procedures

The purpose of traffic control procedures for ATM is to achieve good ATM network efficiency and meet the quality of service requirements by users, also with an approach that is generally applicable. More and more sophisticated traffic control and resource management actions are being developed and applied to practical circumstances.

The basic problem of ATM networks is the statistical behavior of the cell arrival process (e.g. at a buffer where cells generated at several different sources are multiplexed together). It has been found that the parameters of service quality, such as jitter and loss probability, are very sensitive to the assumed resource characteristic. Therefore, it is necessary to use detailed source traffic models for performance evaluation.

Network Resource Management

When several virtual channels are grouped together into a virtual path, only the collective traffic of an entire virtual path has to be handled. In this situation, some other forms of control, such as connection admission control, usage parameter control and network parameter control, can be simplified.

Connection Admission Control

During the call set-up or re-negotiation phase, connection admission control is going to check whether new virtual path or virtual channel can be accepted by the network. That is, if sufficient network resources are available to establish the end-to-end connection, the required quality of service could be met, and the quality of service for any of the existing channels must not be affected by new connection, this new connection will be established.

Usage Parameter Control and Network Parameter Control

Usage parameter control (UPC) and *network parameter control* (NPC) have the same functions but at different interfaces. UPC works at the user network interface, and NPC is applied at the network node interface. The UPC/NPC is used to prevent the degradation of the service quality for established connections in network resources from malicious and unintentional misbehavior.

Priority Control

Each ATM cell is assigned with an explicit cell loss priority bit in the header. Moreover, for a single ATM connection, both priority classes can take an action for different parts of the information with classified into more and less important.

Traffic Shaping

Traffic Shaping is an option for both network operators and users, it helps to dimension network in the way of more cost effective. Traffic shaping actively alters the traffic characteristics of a stream of cells on a VPC or VCC, so that the peak cell rate and cell delay variation are reduced, the burst length is limited.

Congestion Control

Due to unpredictable statistical fluctuations of traffic flows or a network fault, congestion can be caused to overload the network resources. Congestion control is developed to minimize congestion effects and prevent them from spreading. It can be applied at connection admission, or usage parameter control and network parameter control.

Virtual Channels/Paths

There are two types of transport connection in ATM network, virtual paths and virtual channels. A virtual channel is a unidirectional pipe made up from the concatenation of a sequence of connection elements. A virtual path consists of a set of these channels.

Each channel or path is assigned with an identifier with it. All channels within a single path must have distinct channel identifiers, meanwhile, may have the same channel identifier as channels in different virtual paths. An individual channel can be therefore uniquely identified by its virtual channel and virtual path number.

If the connection is switched at somewhere within the network, the virtual channel and path numbers may differ from source to destination. Throughout the connection and staying in a single virtual path, virtual channels will have identical virtual channel identifiers at both ends. Cell sequence is also maintained through a virtual channel connection. Each virtual channel and virtual path has negotiated QoS associated with it.

Virtual Path/Channel Connection:

There are several typical situation in which a virtual channel or virtual path is set up, such as,

- In the case of a permanent or semi-permanent connection, the virtual channel or path may be reserved by the network.
- A new virtual channel connection may be setup within an existing virtual path connection between two user network interfaces.

2.2 Operations Research

OR is employed to solve the complex problems about how to cooperate the operations within large systems of man, machines, material and money, and it has been applied in wide range of applications, such as manufacturing, transportation, construction, telecommunication, financial planning, health care, military and public service. Generally speaking, OR uses the scientific method to investigate and solve the concerned problems.

Typically, the practical problems are carefully observed and formulated, then a scientific (mostly mathematical) model is constructed to represent the important and essential factors of the concerned situation sufficiently and precisely. It attempts to solve and balance the conflicts of interest among the factors so that, in a way, they are best or optimal for overall system [4].

2.2.1 Mathematical Programming

In an optimization problem or mathematical programming, and the most common type of its application, we concern about how to allocate limited resources among the conflicts of activities in an optimal or best manner. Mathematically speaking, we are trying to seek minimize or maximize a function or variables, meanwhile, subject to constraints on the variables. Particularly, "programming" does not refer to computer, in fact, it is essentially in the sense of planning. Also, "a best" instead of "the best" solution is used to imply that there may be multiple choices tied as best [1][4].

2.2.2 Linear Programming

The term "linear programming" defines a particular type of programming application to involve allocating limited resources to activities so that they meet certain value of the overall measure of performance [1]. Common terminology for linear programming model could be summarized as below,

Minimize (or maximize):

$$C^T x$$

Subject to:

$$A x = B$$

$$x \geq 0$$

Where x is the vector of variables to be solved for, A is a matrix with known coefficients, and C and B are vector with known coefficients. The function being minimize (or maximize) is called the objective function, and the equations are referred to as constraints. Particularly, $x \geq 0$ is called non-negativity constraints.

2.2.3 Stochastic Programming (SP)

As stated as above, there is not only linear structure in the linear program, but also the coefficients matrix in *A, B, C* are fixed known data during the whole planing process. However, it is not true in most practical situation [20][6]. In the model of prototype network, the linear program represents a network traffic planing problem: *B* is the coefficients vector of demand external traffic flows, *A* is the coefficients matrix of network path design, *C* is the vector of costs per unit traffic flow and *x* is the vector of optimal traffic flow in paths. In this situation, external traffic flows are typically random variables and, meanwhile, *x* shall be determined optimally. Obviously, in many practical problems, some or all coefficients are random variables instead of fixed and known data [11][12][14].

There are three possibilities' situations:

- These data are random variables with known probability distributions.
- These data are random variables without known probability distributions.
- These data are simple variables.

At the starting point of stochastic linear programming problem, we may think about replacing the random variables in linear program by their expectation values or, good estimation of them, and then solving the relevant linear program.

Assume such a problem is

Minimize:

$$x_1+x_2$$

Subject to:

$$ax_1 + x_2 \geq 5$$

$$bx_1 + x_2 \geq 6$$

$$x_1 \geq 0,\ x_2 \geq 0$$

Where *a* and *b* are uniformly distributed random variables within the [1,3] and [2,4] respectively. Then, the average of *a* and *b* are *2* and *4* respectively. Thus, we could have

Minimize:

$$x_1+x_2$$

Subject to:

$$2x_1 + x_2 \geq 5$$

$$4x_1 + x_2 \geq 6$$

$$x_1 \geq 0,\ x_2 \geq 0$$

Which yield the optimal solution (integer)

$x_1^* = 1$ and $x_2^* = 2$

Meanwhile, we have concern about what is probability that this solution is feasible with respect to the original problem? We get,

$$P\{(a,b)|\ ax_1^* + x_2^* \geq 5\ ;\ bx_1^* + x_2^* \geq 6\}$$

$$=P\{(a,b)|\ 2x_1^* + x_2^* \geq 5\ ;\ 4x_1^* + x_2^* \geq 6\}$$

$$=P\{(a,b)|\ a \geq 2\ ;\ b \geq 4\}$$

$$=(0.5)(0.5)$$

$$=0.25$$

So this solution is infeasible with probability *75%*. Therefore, one should be careful when using the above procedure in view of the possible practical consequences of infeasibility. Actually, in many practical situations, obviously it is not allowed that the above approach is applied.

There are two types of models in SP situation. One of the SP methods is the "Wait and See" model. It concerns the situation, in which the decision variable *X* is chosen after making an observation for the realization of the random variables. This means that a set of samples is realized. Initial interest of the WS approach was to find the PDFs of the optimal solution or the optimal objective function against their stochastic coefficients. Furthermore, practice and experience have shown that, in some applications, the derivation of PDFs is intractable. Thus, sometimes the moment of stochastic variables are sought instead.

For example, in our above model, a complete set of samples of a and b are chosen as the assumption on realization of random vector A, B and C, and then applied to obtain the optimization solutions. Then, statistical characterizations, particularly on probability distribution, of optimal solutions will be main concern in the final part of this procedure.

2.2.4 Stochastic Programming and B-ISDN Optimization

In conventional network studies, the statistical characterizations of traffic arrivals are regularly considered without significant change over time. As a result, network performance could be evaluated in term of average values. One of the typical examples is the average delay of all data packages over a channel. Moreover, other parameters in network model are also described as mean values, such as the average arrival rate, the average inter-arrival time, or the average cost. However, as more and more new technologies are emerging, these conventional approaches seem to be going without enough matching.

As a matter of fact, ATM in B-ISDN provides a typical example. ATM network technology is developed to provide various application services with diverse rates such as constant, variable, available, and unspecified bit rate (CBR, VBR, ABR, and UBR). In some applications, it is required for the moderate bandwidth, such as 2 Kbps, and some others may reach the need of several Mbps or more. Moreover, based on different demands, a variety of *Quality of Service* (QoS) may be significantly different among the clients and also have to be matched [6][1][21]. More advanced features, such as congestion control and self-healing, may introduce additional complexity [12].

In short, modern traffic flows among communication network are either dynamic or stochastic, or both. In such a situation, the traditional approach using mean value of models is becoming invalid. The stochastic model (such as SP problem) is considered more effective to match current practical development. Thus, SP approach is integrated and applied to further network optimization problems. This is also one of the important contributions in this thesis.

2.3 Tele-traffic Modeling

As a branch of telecommunication engineering, tele-traffic modeling is being developed to characterize and elucidate the traffic flow information in communication network. The conventional concept of tele-traffic and relevant topics were developed by H.K Erlang to investigate and evaluate quantitatively the performance of telephone systems, such as the statistics of calls, the expected number of call arrivals per unit time and the characterizations of peak traffic etc. It has been extended to the application of the engineering design problems on exchanging, switching and routing.

The initial consideration on the model of telephone traffic is essentially statistical, or say, the telephone service is dealing with various processes as random events, such as call duration, call arrival rates and the number of calling from subscribers. Tele-traffic model and relevant experience have been successfully applied in the practical design problems on exchanging and switching, and is also proved to be efficient solutions for typical design problems such as the subscriber of telephone service with minimal blocking probability and satisfied voice quality with low delay. Since the major concern in tele-traffic modeling of other telecommunication is still about the statistical characterizations of the involved variables, Erlang's model has been extended and subsequently made compatible with the modern applications, such as computer and data communication or integrated digital transmission of heterogeneous in B-ISDN.

Thus, the following section is going to deal with the stochastic description and modeling consideration specific to B-ISDN telecommunication [6][20].

As the discussion on the stochastic aspects of the tele-traffic, the concept of queuing theory is developed to model, analyze and evaluate real telephone systems as well as various telecommunication systems and their traffic patterns, the performance of information transfer and network design etc.

The important concept in queuing theory is “waiting line”, based on stochastic theory. It is applied to study the customer arrival sequences at service facility as random events. The following are definitions and fundamental quantities commonly used.

Queue: a waiting line of units demanding service at a service facility
Customer: The unit demanding the service
Server: The entity/person who provides the service

Notations
N: Average number of customers in the system
T: Mean time that a customer spends in the system
λ: Mean arrival rate, or average number of customers arriving at the input process per unit time at service point

2.3.1 Little’s Theorem

Average number of customer in the system is equal to Arrival rate $\times$ Average length of time staying in the system. That is,
$N=\lambda T$

2.3.2 M/M/1 Queuing System

M/M/1 is popular in describing the telecommunication queuing system [18][6][20]. This queuing system is simple to analyze and therefore has been adopted extensively to specify the tendencies in B-ISDN network systems. It consists of single queuing station with single server (or say single transmission line), and has negative exponential or memoryless arrivals at a rate of λ arrivals per seconds and random length of arrival entity with a negative exponential distribution.

2.3.3 Arrival Process

In M/M/1, arrival process is used to describe the statistics of customer arrival process in queuing theory. Thus, in the application of telecommunication, such an arrival process

may be the arrival of telephone calls, packet message, or ATM cells. The M/M/1 queueing system includes a single queueing station with a single server. The first letter indicates the nature of the arrival process, M stands for memoryless, which here means a Poisson process. The second letter indicates the nature of the probability distribution of the service times, M stands for exponential. The last number indicates the number of servers.

Frequently, the arrival process is characterized as the Poisson process:

Probability of arrival event in the time interval $\Delta t \rightarrow 0$ is equal to $\lambda \Delta t < 1$,

where λ is a constant and it depicts the arrival rate. Probability of no arrival in Δt is $1-\lambda \Delta t$.

The arrival is memoryless. In a given time interval of length Δt, the arrival event is independent of events in previous and future events.

The number of arrivals in any interval of length τ is Poisson distribution. If $A(t)$ is a counting process that represents the total number of arrivals that have occurred from 0 to t, and for $s < t$, $A(t) - A(s)$ equals the numbers of arrivals in the interval $(s, t]$. For all t and $\tau > 0$, the probability distribution function is given by

$$P\{A(t+\tau) - A(t) = n\} = e^{-\lambda\tau} \frac{(\lambda\tau)^n}{n!}, n = 0,1,...$$

Interarrival times are independent and exponentially distributed with parameter λ; let t_n denote the time of nth arrival, the interval $\tau_n = t_{n+1} - t_n$ follows probability distribution,

$$P\{\tau_n \leq s\} = 1 - e^{-\lambda s}, s \geq 0$$

And the corresponding probability distribution function is,

$$\rho(\tau_n) = \lambda e^{-\lambda \tau_n}$$

Where the mean of τ_n is $1/\lambda$ and variance $1/\lambda^2$.

Actually, the probability distribution function about traffic rate λ_n instead of τ_n is more important to our B-ISDN traffic modeling. Therefore, the following section is to find the probability distribution function about λ_n, based on τ_n.

Since

$$\lambda_n = \frac{1}{\tau_n} \rightarrow \tau_n = \frac{1}{\lambda_n}$$

Also

$$0 \le \tau_n \le s \Rightarrow 0 \le 1/\lambda_n \le s$$

Then

$$\frac{d\tau_n}{d\lambda_n} = -\frac{1}{\lambda_n^2}$$

Thus

$$p(\lambda_n) = \lambda \cdot e^{-\lambda \cdot \left(\frac{1}{\lambda_n}\right)} \cdot \left| -\frac{1}{\lambda_n^2} \right| = \frac{\lambda}{\lambda_n^2} \cdot e^{-\frac{\lambda}{\lambda_n}}$$

Later, the Poisson process described by this formula will be integrated and applied to our further numerical simulation in case studies.

2.4 Network Models

To solve problems of data transmission in network environment involves the main consideration about the design of communication network. There are currently various types of networks with quite different structure and sophistication, such as more common networks that carry a variety of operations in opening or unstructured environment, or the particular type of networks to deal with specific tasks in a carefully managed environment. Continuous progress has been made in this technology, most of the design methodology has been developed with the packet-switched network, but the principles can be extended to more general networks. Moreover, in a common sense, network users are sharing not only processing facilities, but also communication facilities. Thus, the cost-effective configuration and usage of communication channels is the main concern of our further research.

We are going to evaluate the performance of communication network, since the performance models are formulated based on the traffic arrival rates at the network links and paths, these models are named as flow models [1][18][6].

Typical communication network could be modeled by a graph $G = (V, A)$, where V is the set of nodes, and A the set of links. Thus, for most of the packet communication networks, there is an augmented graph with two entities: link traffic flow λ_i and link transmission capacity C_i. A typical example of λ_i is the traffic arrival rate of link i, expressed in data units per second. Ci has the same unit as λ_i.

Network models usually involve two indices: operational index and capital index. One of the typical meaning for operational index is the delay T of packets or the number of packets in the system, and for capital index is the cost D per unit capacity. Then, for typical network optimization problem, these two indices may be combined as functions for either capacities or flows, or both, also involved entities are adjusted and called design variables. Moreover, the typical situation about constraints is that link flows are not greater than link capacities.

In short, for general network models, we mainly consider a performance measure T, a cost index D, and two design variables: channel capacity C_i and channel flow λ_i. It is also necessary to define that, here "channel" could be of either physical or virtual meaning. By given network topology, external traffic flow requirements, and assumption that the flows satisfy the link channel capacities, three specific models can be formulated as follows:

Capacity Assignment (CA) Problem

Given	λ_i *and Topology*
Minimize	T
Adjust	C_i
Constraint	D

Flow (Routing) Assignment (FA) Problem

Given	C_i *and Topology*
Minimize	T
Adjust	λ_i

Capacity and Flow Assignment (CFA) Problem

Given	*Topology*
Minimize	T
Adjust	C_i *and* λi
Constraint	D

The formulations presented above can be implemented using different approaches in terms of the mathematical treatments, which are employed in the graph theory.

Although capacity constraints may not be necessary in the FA formulation, if the objective function includes the average packet delay derived by the M/M/1 queuing model, in practice such a constraint may still need to be explicitly included in the formulation. We have to consider two reasons based on the implementation of the optimization algorithms: if the starting point is not a feasible one, then we can not enter the feasible region; even if starting point is a feasible one, the boundary of the feasible region may be punctured during the numerical searching operations.

In the formulations of the CA and CFA problems, the roles of T and D can be exchanged, thus the terminology of primary and dual will be induced. Whether a model is CA, FA, or CFA, depends on what the design variables are, rather than what variables are presented in the objective function. For example, the FA, CA, and CFA may have the identical formulation of the objective function. However, an additional constraint is necessary, the capacity of upper boundary for the latter two problems. In general, enough attention should be paid to the boundary when the formulation of the objective function is adapted from another model. An objective function should be able to reflect the effect of design variables appropriately in computation. In general, all design variables should be

integrated into the objective function, even if it may not be necessary from analytical reasons.

Conventional formulations of all three generic models are based on the link-node incidence. However, link flows may further be expressed by path flows. Models based on the path-node incidence may have another important property: the Poisson assumption is reasonably applicable. The average packet delay, usually based on the Poisson assumption, has mostly been selected as the performance measure T. But it has recently been indicated in the literature that the Poisson assumption may have to be more carefully investigated for many modern networks[22].

2.5 The Formulation of Virtual Path Optimization

ATM is connection-oriented transfer mode in a packet switched network. Two types of connections are specified by the ATM standard:

- An end-to-end connection will be set up before the communication starts, and is named as VC connection. Usually VC is established based on per call basis.
- The performance of network will be improved, the connection sharing common paths are grouped into a single unit, called VP.

VP concept is developed based on a few motivations such as reducing the cost of network control for establishing and switching VCs by utilizing the exiting VPs, and grouping more connections to less VPs. Therefore, most researches as well as our current topics have been actively focusing on VP management.

Additionally, there are several categorizations about VP lifetime duration or time frame, such as permanent, semi-permanent, and dynamic VPs. We are going to pay main attention on semi-permanent VPs since they are responsible for supporting the general user-to-user traffic. Hereafter, VPs refer to the semi-permanent VPs.

A careful selection of VPs may result in a VP layout that provides a framework of available VCs with low setup and switching cost, and a network, which is resilient to

unexpected traffic loads. However, due to the traffic complexity supported by ATM, the selection of VPs has been proved to be a difficult task. The mathematical expression of the optimization models in terms of VPs is similar to the elementary models described in previous sections.

2.6 Generic Network Models

As we discussed in the previous sections, for general network models, we consider a performance measure T, a cost index D, and two design variables: channel capacity C_i and channel flow λ_i.

For any given network topology and external traffic flow requirements, we assume that the flows satisfy the link channel capacities. Typical FA problem can be formulated as follows:

Flow (Routing) Assignment (FA) Problem

Given	C_i *and Topology*
Minimize	T
Adjust	λ_i

where the capacities are given and the flows of traffic must be determined so as to minimize the performance measure T. Therefore it is expected and necessary to provide more than one path for the traffic flow assignments.

This FA model requires that we minimize the function T with respect to the flows λ_i in such a way that the external traffic flow requirements are satisfied, meanwhile it is still under the assumption of given capacity assignment. Furthermore, the usual conservation of flow law at each node basis must be obeyed. In other words, total traffic flow into any node must equal to the total traffic flow out of that node. As for capacity constraint on each channel, it requires that flows on any channel must be less than the capacities. Thus, such an interesting and obvious property can be found, that the performance function T will be increased whenever the set of flows approach the capacity of their channels. Therefore, in the terminology of mathematical programming, the performance function T

includes the necessary capacity constraint as a penalty function. During the application of minimization techniques, this important property guarantees the feasibility of the solution with respect to the capacity constraint.

Conventional formulations of generic models presented above are based on the link-node incidence. However, in B-ISDN applications, there are multiple classes of traffic like CBR, VBR, UBR and ABR flows to be transmitted over paths connecting a set of OD pairs. Therefore, there is a need to further express link flow λ_i by path flow x_k associated with traffic classes. This is feasible because usually the total number of x_k is greater than the total number of λ_i. Accordingly, the design variables become x_k, and the induced model is based on the path-node incidence. Mathematically, a network optimization problem with the path-node incidence can be described by the MC model, a representative paradigm developed in the theory of network flows [1]. For a B-ISDN paradigm, the concept of commodity classes is associated with traffic classes.

Models based on the path-node incidence may have another important property: the Poisson assumption is still reasonably applicable. This is because a path is typically an end-to-end connection and several types of traffic can be described by the Poisson pattern at the connection layer.

Thus, based on above considerations particularly in B-ISDN applications, general "FA problem" can be modified and then formulated as follows,

Flow Assignment Problem

Given

C_i and Topology

Or say, subject to

$$\sum_{p \in P_w} x_p = \gamma_w$$

$$\sum_{p \in Q_i} x_p \leq C_i$$

$$x_p \geq 0 \ \forall p \in P_w, w \in W$$

Minimize

$$y = \sum_{w \in W} \sum_{p \in P_w} c_p x_p$$

Adjust x_i

W: the set of all OD pairs.
P_w: the set of all paths that connect a particular OD pair w.
Q_i: the set of all paths that pass link i.
x_p: the flow of path p.

2.7 Sensitivity Analysis

SA can be carried out in one of these typical procedures,

- There are two variations in this model that are reported to be invariable: objective function (minimal cost in our case) and constraint conditions (path capacity in our case). Individual coefficient of objective function (path cost or link cost in our case) can vary but without changing the basis associated with an optimal solution. There will be a range of objective function coefficients over which the values of decisions variables, in an optimal solution, will remain unchanged.
- Similarly, if individual constraint value can vary but change nothing on the basis associated with an optimal solution, there will be a range of constraint values over which the values of decision variables, in an optimal solution, will remain unchanged.

Thus, in such a manner, when available resources are not balanced properly or not in the most effective allocation, sensitivity analysis can be used to investigate if additional resources should be acquired to break through possible bottleneck, and give us insight to possible improvements in modeled problem.

2.8 Simulation Model & Case Study on Small Network

In the following, the WS approach is applied to a FA model through the first case study of the prototype network shown in Figure 2-1. Changing individual constraint value among constraint set 1 (on external traffic) is applied to the WS procedure.

The parameters used in management optimization are listed below:

OD pairs: (1, 2), (1, 3), (2, 4), (3, 4)

Link Capacities: 7.0, 7.0, 8.0, 8.5, 8.5 (for links (1, 2), (1, 3), (1, 4), (2, 3), (3, 4), respectively)

Link Costs: 4, 4, 6, 5, 7 (for links (1, 2), (1, 3), (1, 4), (2, 3), (3, 4), respectively)

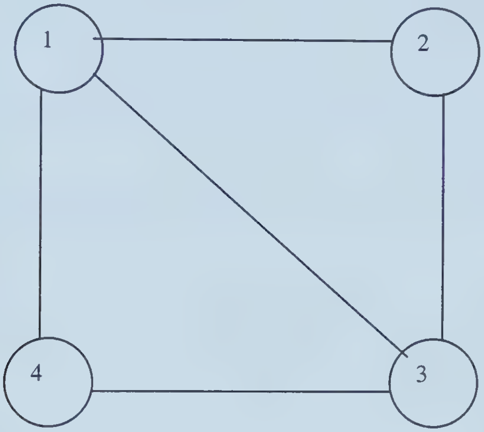


Figure 2-1: Prototype Network

Table 2-1: OD Pairs, Paths and Their Costs

OD Pair Index	OD Pair	Path Index	Path and Nodes	Path Cost (Distance)
1	(1,2)	1	{1,2}	4
		2	{1,3,2}	9
		3	{1,4,3,2}	18
2	(1,3)	4	{1,3}	4
		5	{1,2,3}	9
		6	{1,4,3}	13
3	(2,4)	7	{2,1,4}	10
		8	{2,3,4}	12
4	(3,4)	9	{3,4}	7
		10	{3,1,4}	10
		11	{3,2,1,4}	15

Notation:
External traffic flow: γ_k
Link capacity: C_j
Link cost: e_l
Path flow: x_i
Path cost: c_i (Cost per unit path flow)

Table 2-2: Link Costs and Path Costs

	e_1	e_2	e_3	e_4	e_5	Path Cost
c_1	*					4
c_2		*		*		9
c_3			*	*	*	18
c_4		*				4
c_5	*			*		9
c_6			*		*	13
c_7	*		*			10
c_8				*	*	12
c_9					*	7
c_{10}		*	*			10
c_{11}	*		*	*		15
Link Cost	4	4	6	5	7	

Remark: The path passes through the links marked with "*". The cost of path will be the total cost of links passed by this path.

Minimize: (Cost function in cost per unit path flow)

$$D = c_1 \times x_1 + c_2 \times x_2 + c_3 \times x_3 + c_4 \times x_4 + c_5 \times x_5 + c_6 \times x_6 + c_7 \times x_7 + c_8 \times x_8 + c_9 \times x_9 + c_{10} \times x_{10} + c_{11} \times x_{11}$$

Constraint Set 1:

$$x_1 + x_2 + x_3 = \gamma_1$$

$$x_4 + x_5 + x_6 = \gamma_2$$

$$x_7 + x_8 = \gamma_3$$

$$x_9 + x_{10} + x_{11} = \gamma_4$$

Constraint Set 2:

$$x_1 + x_5 + x_7 + x_{11} \leq C_1$$

$$x_2 + x_3 + x_5 + x_8 + x_{11} \leq C_2$$

$$x_3 + x_6 + x_8 + x_9 \leq C_3$$

$$x_3 + x_6 + x_7 + x_{10} + x_{11} \leq C_4$$

$$x_2 + x_4 + x_{10} \leq C_5$$

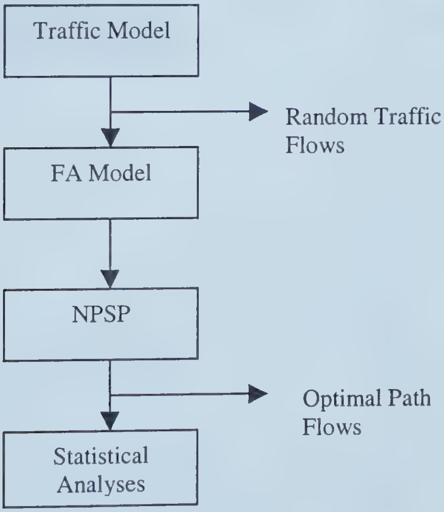
All possible paths were considered for specified OD pairs except paths 2-1-3-4 and 2-3-1-4 for an arbitrary reason. Accordingly, there are 11 paths, where traffic demands

randomly change between 0 and 2.5, with 64 scenarios. The PDFs of external traffic flows are graphically shown in Figures 2-3,7,11 and 14.

Consequently, based on the random traffic model and the FA model of the prototype network, the numerical optimization problems are solved by using *a* nonlinear programming software package developed by Dr. Xian Liu. The relevant mathematical algorithm and software structure are described in Appendix 2. Finally, according to the WS approach, the statistical analyses are applied to the traffic models at OD pairs and the optimal solutions of path flow assignments. This analysis procedure is also summarized into the flow chart below.

The PDFs of optimal path flows are also graphically shown in Figures 2-4,5,6,8,9,10, 12,13,15,16 and 17.

Figure 2-2 : The Flow Chart of Analysis Procedure



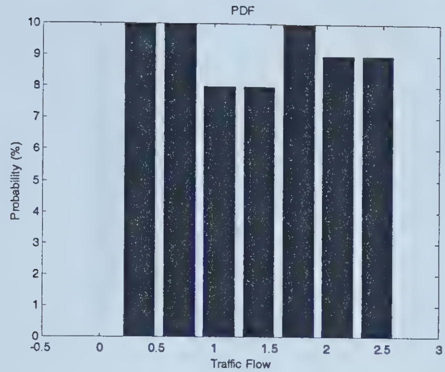


Figure 2-3: External Traffic Flow in OD Pair 1

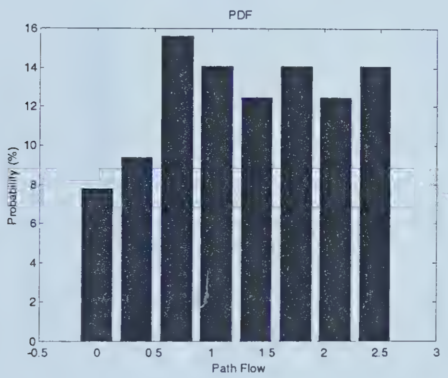


Figure 2-4: Path Flow 1 in OD Pair 1

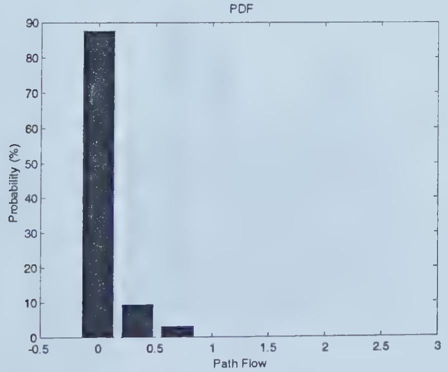


Figure 2-5: Path Flow 2 in OD Pair 1

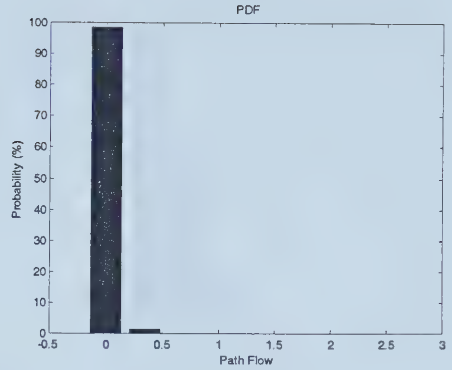


Figure 2-6: Path Flow 3 in OD Pair 1

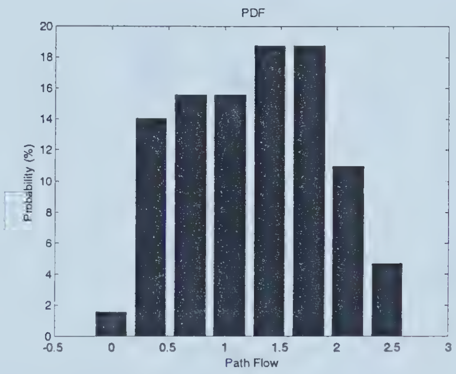


Figure 2-7: External Traffic Flow in OD Pair 2

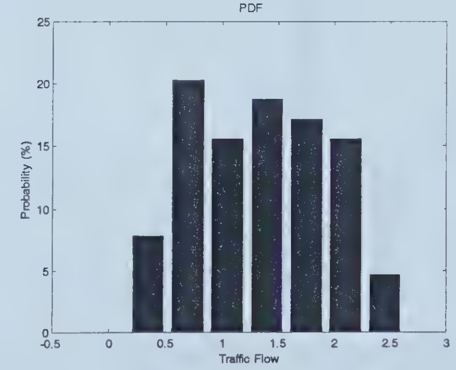


Figure 2-8: Path Flow 4 in OD Pair 2

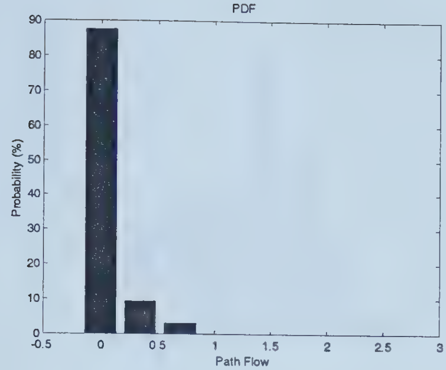


Figure 2-9: Path Flow 5 in OD Pair 2

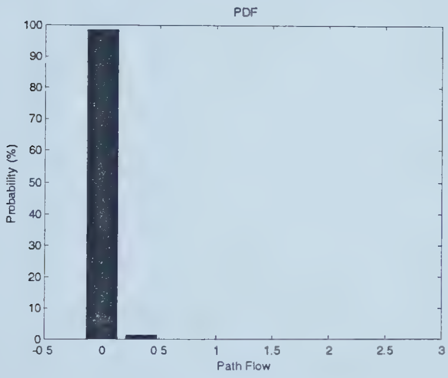


Figure 2-10: Path Flow 6 in OD Pair 2

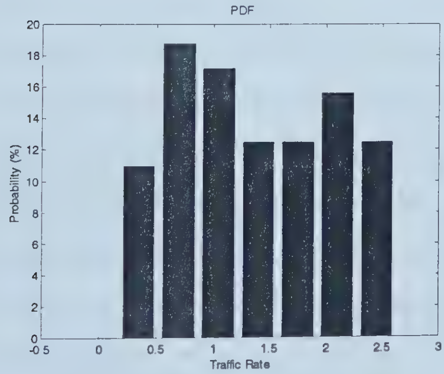


Figure 2-11: External Traffic Flow in OD Pair 3

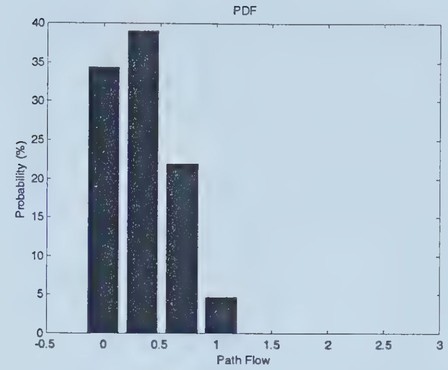


Figure 2-12: Path Flow 7 in OD Pair 3

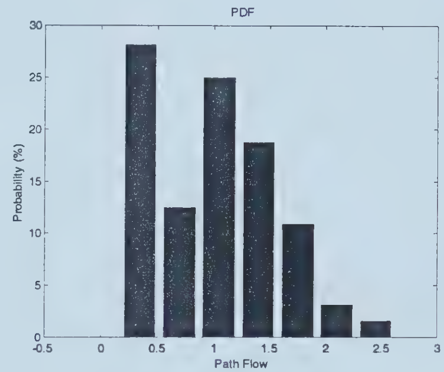


Figure 2-13: Path Flow 8 in OD Pair 3

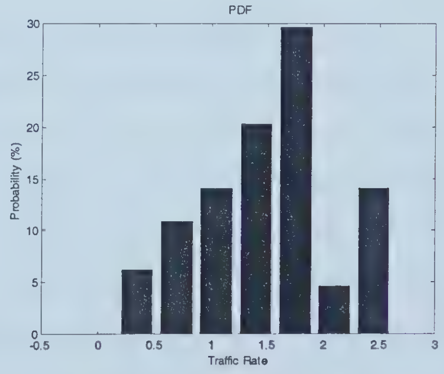


Figure 2-14: External Traffic Flow in OD Pair 4

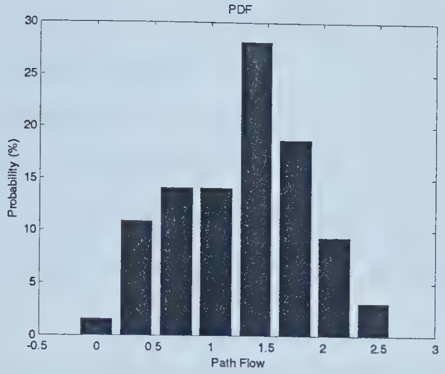


Figure 2-15: Path Flow 9 in OD Pair 4

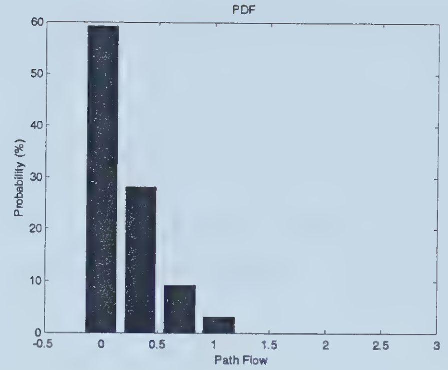


Figure 2-16: Path Flow 11 in OD Pair 4

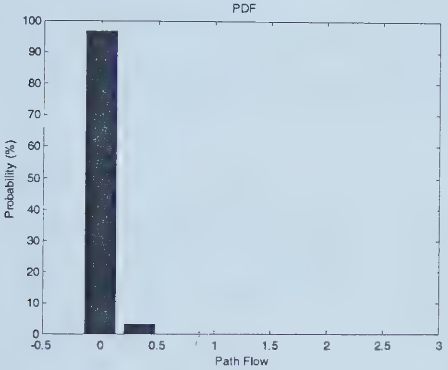


Figure 2-17: Path Flow 10 in OD Pair 4

In all four OD pairs, there are very similar PDFs between the cheapest (shortest) path flows and the external traffic flow into that OD pair. For example, there are similar PDFs between path flow 1 and the external traffic flow into OD pair 1, path flow 4 and the external traffic flow into OD pair 2 and so on. It implies, under optimal flow solutions, the cheapest (shortest) paths will be assigned to transport more traffic flows. This is actually the existing conclusion in the shortest path theory, namely, the flows will go along the shortest (cheapest) if there are multiple paths available and the capacities are large enough.

2.9 A Middle Size Network

Next, WS and SA approaches are applied to a FA model through the second case study of a real network of United States in Figure 2-17. The parameters used in management optimization are listed in below:

Original Network:

28 Nodes and 41 Links

Capacity: all links are assigned with the same unit, 100 unit flow per unit time.

Cost per unit flow and distance: The cost per unit flow is based on the distance, or say, the longer distance refers to the higher unit cost of transferring traffic flow.

Network Model in Flow Assignment Problem:

40 OD pairs and 41 links

Capacity of each link is the same as above

Cost of each link is also the same as the above

80 paths

Two paths are assigned for each OD pair. The first one is the backbone to carry out most traffic, and the second path is the backup to route the traffic in case of a large amount of traffic arrivals.

The capacity of each path is equal to the minimal capacity among the links routed by this path. Cost of each path is equal to the total cost of all links routed by this path.

Formulation of Flow Assignment Model

The selected OD pairs, OD pair indexes, path sets and path indexes are listed in the below Table 2-3. The network topology is shown in Figure 2-17. Based on network topology and some considerations on the formulation of FA problem, the FA model is established and includes cost function (objective function), constraint set 1 and constraint set 2.

Notation:
External traffic flow: γ_k
Link capacity: C_j
Link cost: e_l (Cost per unit link flow)
Path flow: x_i
Path cost: c_i (Cost per unit path flow)

Table 2-3: OD Pairs, Paths and Their Costs

OD Pair Index	OD Pair	Path Index	Path vs. Nodes	Path Cost (Distance)
1	(1,2)	1	{1,2}	38
		2	{1,5,6,7,2}	144
2	(2,4)	3	{2,4}	72
		4	{2,7,8,4}	109
3	(4,13)	5	{4,13}	21
		6	{4,8,14,13}	66
4	(17,18)	7	{17,18}	21
		8	{17,16,19,18}	53
5	(1,5)	9	{1,5}	48
		10	{1,2,7,6,5}	134
6	(2,7)	11	{2,7}	40
		12	{2,4,8,7}	141
7	(4,8)	13	{4,8}	16
		14	{4,13,14,8}	71
8	(13,14)	15	{13,14}	15
		16	{13,4,8,14}	72
9	(17,16)	17	{17,16}	20
		18	{17,18,19,16}	54
10	(18,19)	19	{18,19}	15
		20	{18,20,19}	37
11	(18,20)	21	{18,20}	26
		22	{18,19,20}	26
12	(5,6)	23	{5,6}	33
		24	{5,9,6}	75
13	(6,7)	25	{6,7}	23
		26	{6,9,10,7}	144
14	(7,8)	27	{7,8}	53
		28	{7,10,11,12,8}	186
15	(8,14)	29	{8,14}	35
		30	{8,12,15,14}	62
16	(14,16)	31	{14,16}	21
		32	{14,21,16}	31
17	(16,19)	33	{16,19}	18
		34	{16,20,19}	33
18	(19,20)	35	{19,20}	11
		36	{19,18,20}	41
19	(16,20)	37	{16,20}	22
		38	{16,19,20}	29
20	(5,9)	39	{5,9}	33
		40	{5,6,9}	75
21	(6,9)	41	{6,9}	42
		42	{6,5,9}	66
22	(7,10)	43	{7,10}	62
		44	{7,6,9,10}	105
23	(8,12)	45	{8,12}	20
		46	{8,14,15,12}	77
24	(14,15)	47	{14,15}	17
		48	{14,21,15}	33
25	(14,21)	49	{14,21}	15
		50	{14,15,21}	35
26	(16,21)	51	{16,21}	16

		52	{16,14,21}	36
27	(20,23)	53	{20,23}	41
		54	{20,16,21,23}	74
28	(12,11)	55	{12,11}	46
		56	{12,15,21,22,11}	135
29	(21,22)	57	{21,22}	32
		58	{21,23,22}	54
30	(21,23)	59	{21,23}	36
		60	{21,22,23}	50
31	(9,10)	61	{9,10}	40
		62	{9,6,7,10}	127
32	(10,11)	63	{10,11}	58
		64	{10,7,8,12,11}	181
33	(11,22)	65	{11,22}	60
		66	{11,25,26,22}	81
34	(22,23)	67	{22,23}	18
		68	{22,21,23}	68
35	(12,15)	69	{12,15}	25
		70	{12,8,14,15}	72
36	(15,21)	71	{15,21}	18
		72	{15,14,21}	32
37	(11,25)	73	{11,25}	20
		74	{11,22,26,25}	121
38	(22,26)	75	{22,26}	29
		76	{22,23,27,26}	70
39	(23,27)	77	{23,27}	25
		78	{23,22,26,27}	74
40	(25,26)	79	{25,26}	32
		80	{25,11,22,26}	109

Minimize:
(Cost function in cost per unit link flow)

$$\begin{aligned}
D = \; & e_1 \times (x_1 + x_{10}) \\
& + e_2 \times (x_3 + x_{12}) \\
& + e_3 \times (x_5 + x_{14} + x_{16}) \\
& + e_4 \times (x_7 + x_{18}) \\
& + e_5 \times (x_2 + x_9) \\
& + e_6 \times (x_2 + x_4 + x_{10} + x_{11}) \\
& + e_7 \times (x_4 + x_6 + x_{12} + x_{13} + x_{16}) \\
& + e_8 \times (x_6 + x_{14} + x_{15}) \\
& + e_9 \times (x_8 + x_{17}) \\
& + e_{10} \times (x_8 + x_{18} + x_{19} + x_{22} + x_{36}) \\
& + e_{11} \times (x_{20} + x_{21} + x_{36}) \\
& + e_{12} \times (x_2 + x_{10} + x_{23} + x_{40} + x_{42}) \\
& + e_{13} \times (x_2 + x_{10} + x_{25} + x_{44} + x_{62}) \\
& + e_{14} \times (x_4 + x_{12} + x_{27} + x_{64}) \\
& + e_{15} \times (x_6 + x_{14} + x_{16} + x_{29} + x_{46} + x_{70}) \\
& + e_{16} \times (x_{31} + x_{52}) \\
& + e_{17} \times (x_8 + x_{18} + x_{33} + x_{38})
\end{aligned}$$

$$
\begin{aligned}
&+ e_{18} \times (x_{20} + x_{22} + x_{34} + x_{35} + x_{38}) \\
&+ e_{19} \times (x_{34} + x_{37} + x_{54}) \\
&+ e_{20} \times (x_{24} + x_{39} + x_{42}) \\
&+ e_{21} \times (x_{24} + x_{26} + x_{40} + x_{41} + x_{44} + x_{62}) \\
&+ e_{22} \times (x_{26} + x_{28} + x_{43} + x_{62} + x_{64}) \\
&+ e_{23} \times (x_{28} + x_{30} + x_{45} + x_{64} + x_{70}) \\
&+ e_{24} \times (x_{30} + x_{46} + x_{47} + x_{50} + x_{70} + x_{72}) \\
&+ e_{25} \times (x_{32} + x_{48} + x_{49} + x_{52} + x_{72}) \\
&+ e_{26} \times (x_{32} + x_{51} + x_{54}) \\
&+ e_{27} \times x_{53} \\
&+ e_{28} \times (x_{28} + x_{55} + x_{64}) \\
&+ e_{29} \times (x_{56} + x_{57} + x_{60} + x_{68}) \\
&+ e_{30} \times (x_{58} + x_{59} + x_{54} + x_{68}) \\
&+ e_{31} \times (x_{26} + x_{44} + x_{61}) \\
&+ e_{32} \times (x_{28} + x_{63}) \\
&+ e_{33} \times (x_{56} + x_{65} + x_{74} + x_{80}) \\
&+ e_{34} \times (x_{58} + x_{60} + x_{67} + x_{76} + x_{78}) \\
&+ e_{35} \times (x_{30} + x_{46} + x_{56} + x_{69}) \\
&+ e_{36} \times (x_{48} + x_{50} + x_{56} + x_{71}) \\
&+ e_{37} \times (x_{66} + x_{73} + x_{80}) \\
&+ e_{38} \times (x_{66} + x_{74} + x_{75} + x_{78} + x_{80}) \\
&+ e_{39} \times (x_{76} + x_{77}) \\
&+ e_{40} \times (x_{66} + x_{74} + x_{79}) \\
&+ e_{41} \times (x_{76} + x_{78})
\end{aligned}
$$

(Cost function in cost per unit path flow)

$$D = \sum_{p=1}^{80} c_p x_p$$

Constraint Set 1:

$$\gamma_p = \sum_{p=1}^{40} x_{2p-1} + x_{2p}$$

Constraint Set 2:

$x_1 + x_{10} \leq C_1$

$x_3 + x_{12} \leq C_2$

$x_5 + x_{14} + x_{16} \leq C_3$

$x_7 + x_{18} \leq C_4$

$x_2 + x_9 \leq C_5$

$x_2 + x_4 + x_{10} + x_{11} \leq C_6$

$x_4 + x_6 + x_{12} + x_{13} + x_{16} \leq C_7$

$x_6 + x_{14} + x_{15} \leq C_8$

$x_8 + x_{17} \leq C_9$

$$x_8 + x_{18} + x_{19} + x_{22} + x_{36} \leq C_{10}$$
$$x_{20} + x_{21} + x_{36} \leq C_{11}$$
$$x_2 + x_{10} + x_{23} + x_{40} + x_{42} \leq C_{12}$$
$$x_2 + x_{10} + x_{25} + x_{44} + x_{62} \leq C_{13}$$
$$x_4 + x_{12} + x_{27} + x_{64} \leq C_{14}$$
$$x_6 + x_{14} + x_{16} + x_{29} + x_{46} + x_{70} \leq C_{15}$$
$$x_{31} + x_{52} \leq C_{16}$$
$$x_8 + x_{18} + x_{33} + x_{38} \leq C_{17}$$
$$x_{20} + x_{22} + x_{34} + x_{35} + x_{38} \leq C_{18}$$
$$x_{34} + x_{37} + x_{54} \leq C_{19}$$
$$x_{24} + x_{39} + x_{42} \leq C_{20}$$
$$x_{24} + x_{26} + x_{40} + x_{41} + x_{44} + x_{62} \leq C_{21}$$
$$x_{26} + x_{28} + x_{43} + x_{62} + x_{64} \leq C_{22}$$
$$x_{28} + x_{30} + x_{45} + x_{64} + x_{70} \leq C_{23}$$
$$x_{30} + x_{46} + x_{47} + x_{50} + x_{70} + x_{72} \leq C_{24}$$
$$x_{32} + x_{48} + x_{49} + x_{52} + x_{72} \leq C_{25}$$
$$x_{32} + x_{51} + x_{54} \leq C_{26}$$
$$x_{53} \leq C_{27}$$
$$x_{28} + x_{55} + x_{64} \leq C_{28}$$
$$x_{56} + x_{57} + x_{60} + x_{68} \leq C_{29}$$
$$x_{58} + x_{59} + x_{54} + x_{68} \leq C_{30}$$
$$x_{26} + x_{44} + x_{61} \leq C_{31}$$
$$x_{28} + x_{63} \leq C_{32}$$
$$x_{56} + x_{65} + x_{74} + x_{80} \leq C_{33}$$
$$x_{58} + x_{60} + x_{67} + x_{76} + x_{78} \leq C_{34}$$
$$x_{30} + x_{46} + x_{56} + x_{69} \leq C_{35}$$
$$x_{48} + x_{50} + x_{56} + x_{71} \leq C_{36}$$
$$x_{66} + x_{73} + x_{80} \leq C_{37}$$
$$x_{66} + x_{74} + x_{75} + x_{78} + x_{80} \leq C_{38}$$
$$x_{76} + x_{77} \leq C_{39}$$
$$x_{66} + x_{74} + x_{79} \leq C_{40}$$
$$x_{76} + x_{78} \leq C_{41}$$

2.10 The Traffic Arrival Model and the Poisson Condition

The Poisson model has been considered to be dominant in modern communication development. Particularly, as a classical model, the Poisson model is an appropriate approximation for traffic patterns in virtual circuit switching, such as call connection scenarios and datagram switching in TELNET or FTP applications [6]. Therefore, it is widely recognized with a theoretical significance, and can be helpfully applied to many analytical circumstances.

Accordingly, the Poisson traffic model is integrated into the WS approach and applied to the FA model described through the case study of the Unite States network shown in the previous section. By the definition on the Poisson model in previous section, the PDF on arrival rate λ_n was developed and described as such a formula,

$$\rho(\lambda_n) = \lambda \cdot e^{-\lambda \cdot \left(\frac{1}{\lambda_n}\right)} \cdot \left| -\frac{1}{\lambda_n^2} \right| = \frac{\lambda}{\lambda_n^2} \cdot e^{-\frac{\lambda}{\lambda_n}}$$

Notations

- λ: Mean arrival rate, or average number of customers arriving at the input process per unit time at service point.
- Let t_n denote the time of nth arrival, λ_n is arrival rate of nth arrival.

Chapter 3 Simulation of a Middle-Size Network

3.1 Case Study 1

Based on the network structure and FA model discussed in last chapter, a numerical optimization program was designed to carry out simulation, and the detailed procedure will be discussed in the following sections.

3.1.1 Traffic Arrival Model in Simulation Procedure

Based on the Poisson model introduced in previous section, the PDF on arrival rate λ_n was developed, and then applied to our current traffic models. The relevant parameters about this Poisson model are listed in below.

The traffic demands have sixty-four scenarios and randomly change for 0 and 10 with average of 5. Each scenario has a probability, specified in Table 3-1. The PDF is graphically shown in Figure 3-1.

Table 3-1: Scenarios and Probabilities

Scenario	Value	Probability
1	0.85	1.924%
2	1.05	3.903%
3	1.56	8.336%
4	3.21	10.215%
5	3.81	9.267%
6	5.07	7.255%
7	5.26	6.982%
8	5.33	6.890%
9	5.35	6.857%
10	5.64	6.475%
11	6.31	5.683%
12	6.64	5.338%
13	6.76	5.220%
14	7.80	4.327%
15	8.21	4.034%
16	9.01	3.534%
17	9.07	3.505%

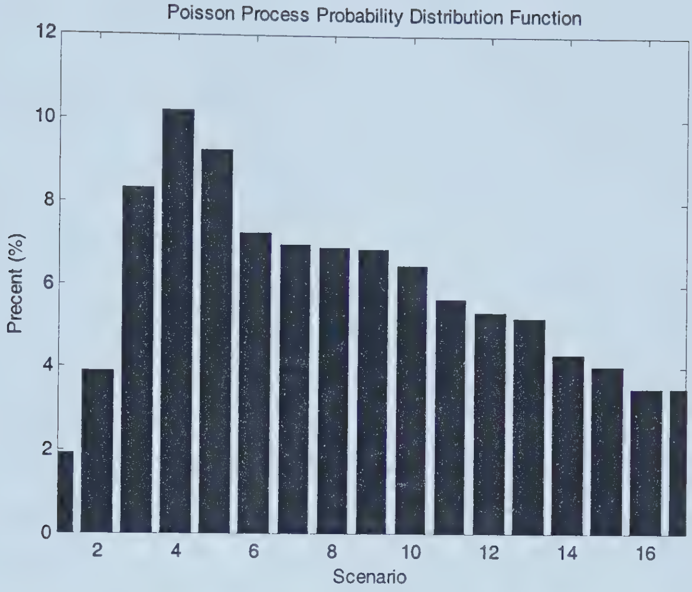


Figure 3-1: PDF in Poisson Process

It is assumed that, at the same time, external traffic flows into all OD pairs follow exactly the same PDF. But, at individual OD pair, sixty-four scenarios are arranged in totally different (and random) sequences.

The behavior of external traffic requirement at OD pair 1 is graphically shown in Figure 3-1. External traffic flows into other OD pairs have same PDF in OD pair 1.

3.1.2 Flow Assignment model and Sensitivity Analysis

The modified FA problem is applied to our network optimization problem, the general FA model is,

Given

C_i and Topology

Or say, subject to

$$\sum_{p \in P_w} x_p = \gamma_w \text{ (Constraint Set 1)}$$

$$\sum_{p \in Q_i} x_p \leq C_i \quad \text{(Constraint Set 2)}$$

$$x_p \geq 0 \ \forall p \in P_w, w \in W$$

Minimize

$$y = \sum_{w \in W} \sum_{p \in P_w} c_p x_p$$

Adjust x_i

W: the set of all OD pairs.

P_w: the set of all paths that connect a particular OD pair w.

Q_i: the set of all paths that pass link i.

x_p: the flow of path p.

A new and modified SA approach was developed in the combined situation of changing both individual coefficient of objective function and individual constraint value. There will be a range on both objective function coefficients and constraint values, over which the values of decision variables (in an optimal solution) will remain unchanged.

Particularly, we are going to investigate the range on changing both objective function coefficients (path cost) and constraint values (capacity and external traffic) over which the values of decisions variables (in an optimal solution) will remain unchanged. Furthermore, as our discussion in the previous section on WS and relevant considerations, the typical concern in a WS problem is statistical moments, *i.e.*, the probability distribution of optimal solutions. Thus, based on the combined approaches of SA and WS, we develop a new measurement model to investigate overall characterizations FA problem. In this measurement model, the statistical characterizations of 'variance of optimal solutions', such as PDF, is detected over the ranges on changing both objective function coefficients (path cost) and constraint values (path capacity and external traffic). This is also the main contribution throughout our research on B-ISDN optimization.

3.1.3 Some Considerations on Path Capacity and Cost

In typical FA model, we assume that the capacities are given and the flows of traffic must be determined so as to minimize the performance measure. Therefore it is expected and necessary to provide more than one path for the traffic flow assignments. In our case, two paths are assigned for each OD pair. The first one is the backbone path in shortest distance (or say lowest cost) to carry out most traffic, and the latter is the backup path in longer distance (or say higher cost) to route extra traffic in case of a large amount of arrivals. Moreover, all backbone paths are across one link only, or say, in the hopefully lowest cost/shortest distance. The backbone and backup paths will work together to carry total traffic flow into relevant OD pair, meanwhile different path traffic will be allocated in an optimal or best manner according to some considerations on their limited resources such as capacity and cost.

Capacity

In a FA model, path capacity and external traffic are considered as constraints for allocating maximum traffic into this path, i.e., external traffic into OD pair can not be beyond the total capacities of the backbone and backup paths.

As our previous description on sensitivity analysis, we are interested in the following situations: over certain ranges of changing path capacities (constraint set 1), external traffic flow (constraint set 2) and cost coefficients, the values of the optimal solution is reported to be invariable or not. Particularly, in our case, this problem is actually about how the optimal path flow is sensitive to the selection of parameters.

In order to present a directly perceivable measurement, we wish to develop a 3D metric. However, for three dimensions of graphic, the maximal three variables can be represented within this figure. In our case, one axis has to be used for indicting variance measurement. The other two are applied to reflect the various realizations of constraints and cost coefficients. If the various realizations of cost coefficients take one of them, then there will be a particular different situation about how to describe two sets of constraints in another axis.

To solve this problem, the concerned factors in terms of the backbone, backup path capacities and maximum external traffic are expressed in such a formula,

$$\alpha = \frac{C_{backbone} + C_{backup}}{Max(\gamma_i)}$$

Such a consideration is about how the total path capacities within the same OD pair handle traffic flow. This definition also applies to more general network model, such as an OD pair with more than two paths, and then can be written as,

$$\alpha = \frac{\sum C_n}{Max\ (\gamma_i)}$$
$$where \quad C_n \in i$$

We consider such a kind of situation, if external traffic is growing and approaches the maximum capacity of any path, how will the optimal path flows be assigned? Will the shortest (cheapest) path get most traffic? Meanwhile, will the longer (more expensive) paths transfer little? The later numerical experiments actually show a quite complicate situation.

In our simulation, based on some considerations discussed as above, the various realizations on the (increased) relevant path capacities and (fixed) maximum external traffic are shown in Table 3-2.

Table 3-2: Path Capacities and Maximum Flows

Realizations	1	2	3	4	5	6	7	8
Backbone capacity	10	15	20	25	30	35	40	45
Backup capacity	10	15	20	25	30	35	40	45
Maximum flows	10	10	10	10	10	10	10	10
α	2	3	4	5	6	7	8	9

Cost

In SA and WS procedure, the various realizations of cost coefficients also interest us. For example, over what kind of range of changing cost coefficients, objective function

(minimal cost in our case) or optimal solution is reported to be invariable. Then, in this case, one of our concerns will be how optimal path flow is sensitive to the selection of the given designs on cost coefficients? For example, for certain (fixed) path capacities or the ratio of α, the optimal (minimal cost) decision on flow assignments may be obtained in such a typical allocation activity for more flow into lower cost path (shortest path theory). Thus, we are going to investigate, under what kind of circumstances, the principle will be able to exist or not. In this particular model, the problem could be expressed in such a formula,

$\beta = c_{backbone}/c_{backup}$

Furthermore, this idea can be applied to more general situation. For example, an OD pair with a backbone path and more than two backup paths. Similarly, the backbone path is used to transmit most traffic, and the backup paths work to route extra traffic in case of a large amount of arrivals. Such a problem is then formulated as a ratio in below,

$$\beta = \frac{Max\ (c_1, c_2, c_3)}{Min\ (c_1, c_2, c_3)} \text{ or } = \frac{Min\ (c_1, c_2, c_3)}{Max\ (c_1, c_2, c_3)}$$

Thus, for different ratio β, how will the optimal path flows be assigned also interest us. Then, in our WS & SA procedure, the various realizations on the (increased) relevant path costs ratio β are shown in Table 3-3.

Table 3-3: Path Costs

Realizations	1	2	3	4	5	6	7	8
Backbone cost	40	40	40	40	40	40	40	40
Backup cost	2.5	5	10	20	40	80	160	320
β	6.25%	12.5%	25%	50%	100%	200%	400%	800%

OD Pair

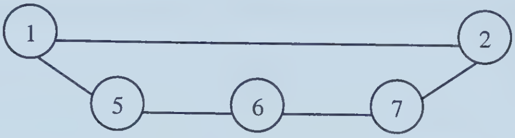
In our case study, all backbone paths pass one link only. However, there are three different types of backup paths. Some pass up to two links, some up to three links, and others up to four links. Three typical instances of OD pairs are selected to investigate the optimal flow solutions between their backbone and backup paths. They are also listed in Table 3-4.

Table 3-4: Example OD Pairs

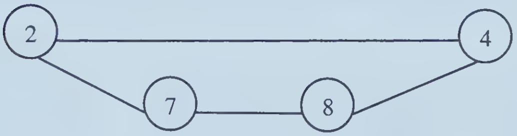
OD pair index	Backbone path index	Backup path index	Backup path number of links passed by the
1	1	2	4
2	3	4	3
11	21	22	2

The graphical representations of OD pairs and relevant paths are shown in below,

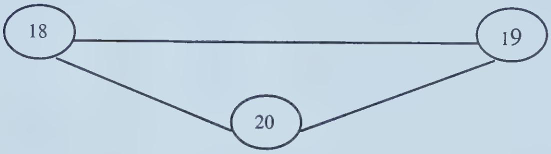
OD Pair 1:



OD Pair 2:



OD Pair 11:



3.1.4 Analysis Procedure

Based on the traffic arrival model and the FA model for US network, the numerical optimization problems are still solved by using the nonlinear programming software package developed by Dr. Xian Liu. Then, the analysis procedure is developed and based on the combined considerations on both the WS approach in SP and the SA method in applied OR.

In this simulation procedure, SA approach is implemented by using the various realizations on eight sets of α (for cost) and eight sets of β (for capacity) simultaneously and then conducting the variance measurements with respect to other considerations on WS.

Under the various realizations of relevant paths and capacities, the estimations of the PDF of optimal flow solutions are the most important processes during WS procedure. Meanwhile, with consideration on SA method, the estimations of PDF can be extended, and then applied to the (8 x 8 = 64 points of) measurements on the variance ratios

between optimal flow solutions in backbone or backup paths, and external traffic flow (64 scenarios).

Since the PDF of demand traffic is known, we are going to estimate the PDF of optimal path flows by following such a procedure. Optimal path flows are classified into individual group by their thresholds. The lower bound of thresholds is calculated based on the mean value of current scenario and pervious one. The upper bound is the mean value of current scenario and next one, and so on. The thresholds are listed in Tables 3-5. Probabilities among individual range are then easily obtained.

Table 3-5: Thresholds List

Thresholds Index	Thresholds Lower Bound	Thresholds Upper Bound	Relevant Probability	Relevant Scenario	Scenario Probability
1	0	0.95	P_1	0.85	1.924%
2	0.95	1.31	P_2	1.05	3.903%
3	1.31	2.39	P_3	1.56	8.336%
4	2.39	3.51	P_4	3.21	10.215%
5	3.51	4.44	P_5	3.81	9.267%
6	4.44	5.17	P_6	5.07	7.255%
7	5.17	5.30	P_7	5.26	6.982%
8	5.30	5.34	P_8	5.33	6.890%
9	5.34	5.50	P_9	5.35	6.857%
10	5.50	5.98	P_{10}	5.64	6.475%
11	5.98	6.48	P_{11}	6.31	5.683%
12	6.48	6.70	P_{12}	6.64	5.338%
13	6.70	7.28	P_{13}	6.76	5.220%
14	7.28	8.01	P_{14}	7.8	4.327%
15	8.01	8.61	P_{15}	8.21	4.034%
16	8.61	9.04	P_{16}	9.01	3.534%
17	9.04	+8	P_{17}	9.07	3.505%

Thus, a particular variance measurement named *Difference Ratio* (DR) is designed to investigate the trend of probability distribution changing under various realizations of (8 x 8=) 64 sets of cost and capacity, and is calculated by using the below formula,

$$Difference\ Ratio = \sqrt{(P_1 - 1.921\%)^2 + (P_2 - 3.097\%)^2 + \ldots\ldots + (P_{17} - 3.505\%)^2}$$

Actually DR is applied to express the degree of similarity between two measured PDFs. Lower DR refers to very similar situation between two measured PDFs. Particularly, if this concept is extended to further FA problems, lower DR states similar PDFs between optimal flow solution and external traffic flow. It implies that the path flow with lower DR is assigned more traffic flow than other paths with higher DR within same OD pair.

This analysis procedure can be summarized as a flow chart in below. For three typical OD pairs, (8 x 8 =) 64 points of trend on DR will be obtained for each path. The original data (on DR) is shown in Tables 3-6,7,8,9,10 and11, and their graphical representations are included in Figures 3-3,4,5,6,7 and 8.

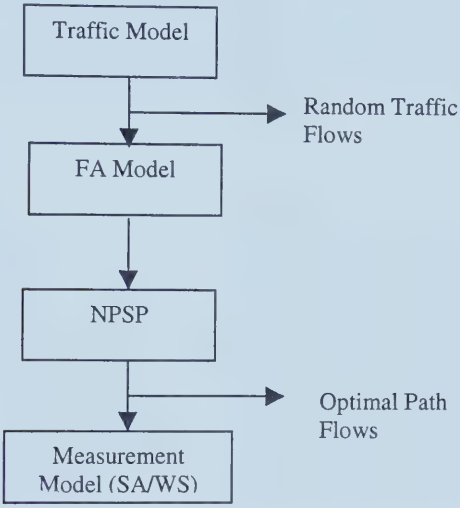


Figure 3-2: The Flow Chart of Analysis Procedure

Table 3-6: Difference Ratio for Path 1 (in OD Pair 1)

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
α_1	0.3047	0.3092	0.3135	0.3221	0.2747	0.1223	0.1307	0.1368
α_2	0.3696	0.3710	0.3977	0.4026	0.3173	0.1307	0.1307	0.1307
α_3	0.3601	0.3865	0.3904	0.3916	0.3203	0.1307	0.1307	0.1307
α_4	0.3733	0.3878	0.3890	0.3870	0.3203	0.1307	0.1307	0.1307
α_5	0.3733	0.3878	0.3890	0.3870	0.3203	0.1307	0.1307	0.1307
α_6	0.3733	0.3878	0.3890	0.3870	0.3203	0.1307	0.1307	0.1307
α_7	0.3733	0.3878	0.3890	0.3870	0.3203	0.1307	0.1307	0.1307
α_8	0.3733	0.3878	0.3890	0.3870	0.3203	0.1307	0.1307	0.1307

Table 3-7: Difference Ratio for Path 2 (in OD Pair 1)

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
α_1	0.5747	0.5626	0.5622	0.5850	0.7485	0.9680	0.9680	0.9680
α_2	0.7941	0.7802	0.7641	0.6325	0.9456	0.9680	0.9680	0.9680
α_3	0.8097	0.7979	0.8097	0.6426	0.9456	0.9680	0.9680	0.9680
α_4	0.8256	0.7822	0.8097	0.6359	0.9456	0.9680	0.9680	0.9680
α_5	0.8256	0.7822	0.8097	0.6359	0.9456	0.9680	0.9680	0.9680
α_6	0.8256	0.7822	0.8097	0.6359	0.9456	0.9680	0.9680	0.9680
α_7	0.8256	0.7822	0.8097	0.6359	0.9456	0.9680	0.9680	0.9680
α_8	0.8256	0.7822	0.8097	0.6359	0.9456	0.9680	0.9680	0.9680

Table 3-8: Difference Ratio for Path 3 (in OD Pair 2)

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
α_1	0.3492	0.3241	0.3583	0.3330	0.2772	0.1307	0.1307	0.1307
α_2	0.3581	0.3673	0.3693	0.3919	0.3068	0.1368	0.1307	0.1307
α_3	0.3476	0.3640	0.3647	0.3971	0.3068	0.1461	0.1307	0.1307
α_4	0.3576	0.3715	0.3698	0.3971	0.3068	0.1453	0.1307	0.1307
α_5	0.3576	0.3715	0.3698	0.3971	0.3068	0.1453	0.1307	0.1307
α_6	0.3576	0.3715	0.3698	0.3971	0.3068	0.1453	0.1307	0.1307
α_7	0.3576	0.3715	0.3698	0.3971	0.3068	0.1453	0.1307	0.1307
α_8	0.3576	0.3715	0.3698	0.3971	0.3068	0.1453	0.1307	0.1307

Table 3-9: Difference Ratio for Path 4 (in OD Pair 2)

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
α_1	0.5899	0.5604	0.5500	0.5447	0.7748	0.9680	0.9680	0.9680
α_2	0.8256	0.6482	0.7463	0.6568	0.9768	0.9680	0.9680	0.9680
α_3	0.8256	0.6550	0.7337	0.6512	0.9768	0.9680	0.9680	0.9680
α_4	0.8256	0.6422	0.7031	0.6386	0.9768	0.9680	0.9680	0.9680
α_5	0.8256	0.6422	0.7031	0.6386	0.9768	0.9680	0.9680	0.9680
α_6	0.8256	0.6422	0.7031	0.6386	0.9768	0.9680	0.9680	0.9680
α_7	0.8256	0.6422	0.7031	0.6386	0.9768	0.9680	0.9680	0.9680
α_8	0.8256	0.6422	0.7031	0.6386	0.9768	0.9680	0.9680	0.9680

Table 3-10: Difference Ratio for Path 21 (in OD Pair 11)

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
α_1	0.3685	0.3678	0.3688	0.3377	0.3019	0.1364	0.1307	0.1307
α_2	0.3728	0.3959	0.3580	0.3905	0.3068	0.1368	0.1307	0.1307
α_3	0.3718	0.3969	0.3523	0.3820	0.2928	0.1307	0.1307	0.1307
α_4	0.3803	0.3921	0.3565	0.3820	0.2965	0.1307	0.1307	0.1307
α_5	0.3803	0.3921	0.3565	0.3820	0.2965	0.1307	0.1307	0.1307
α_6	0.3803	0.3921	0.3565	0.3820	0.2965	0.1307	0.1307	0.1307
α_7	0.3803	0.3921	0.3565	0.3820	0.2965	0.1307	0.1307	0.1307
α_8	0.3803	0.3921	0.3565	0.3820	0.2965	0.1307	0.1307	0.1307

Table 3-11: Difference Ratio for Path 22 (in OD Pair 11)

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
α_1	0.6948	0.6948	0.6711	0.6512	0.7619	0.9680	0.9680	0.9680
α_2	0.8256	0.7979	0.7802	0.6859	0.9308	0.9680	0.9680	0.9680
α_3	0.8256	0.7821	0.7802	0.6561	0.9768	0.9680	0.9680	0.9680
α_4	0.8256	0.7821	0.7803	0.6380	0.9768	0.9680	0.9680	0.9680
α_5	0.8256	0.7821	0.7803	0.6380	0.9768	0.9680	0.9680	0.9680
α_6	0.8256	0.7821	0.7803	0.6380	0.9768	0.9680	0.9680	0.9680
α_7	0.8256	0.7821	0.7803	0.6380	0.9768	0.9680	0.9680	0.9680
α_8	0.8256	0.7821	0.7803	0.6380	0.9768	0.9680	0.9680	0.9680

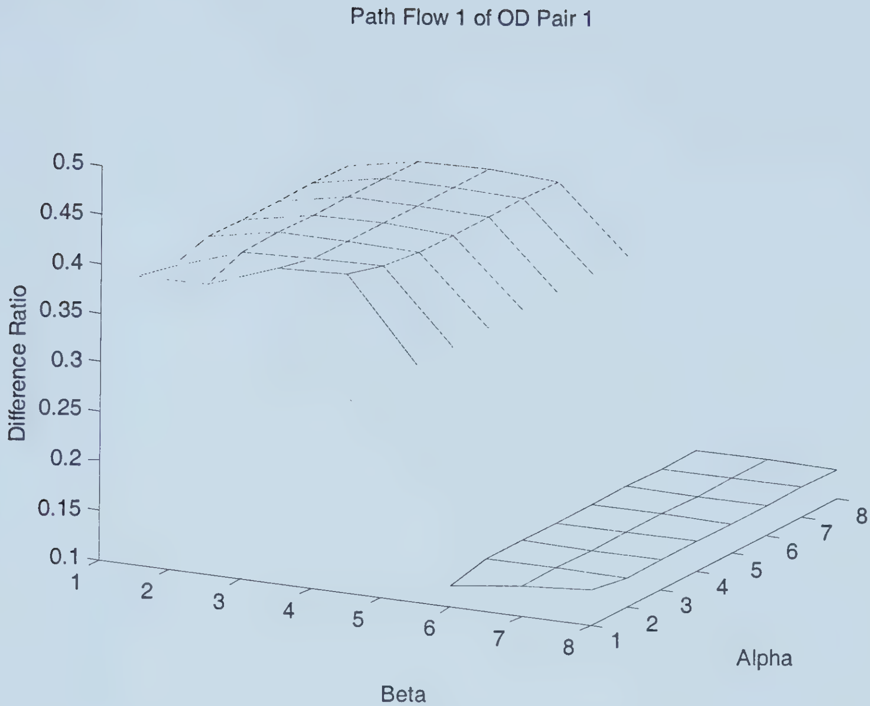


Figure 3-3: Path Flow 1 in OD Pair 1

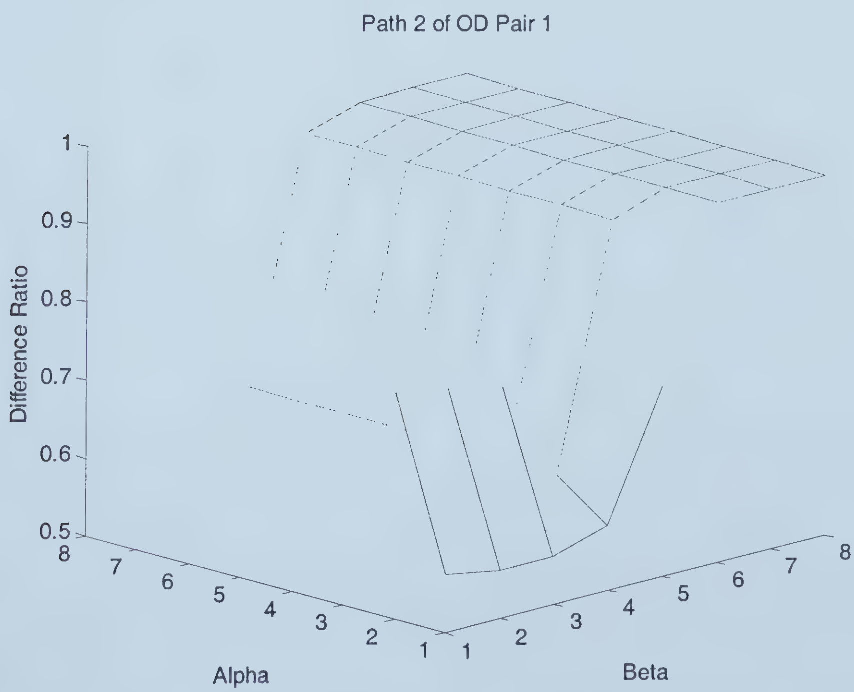


Figure 3-4: Path Flow 2 in OD Pair 1

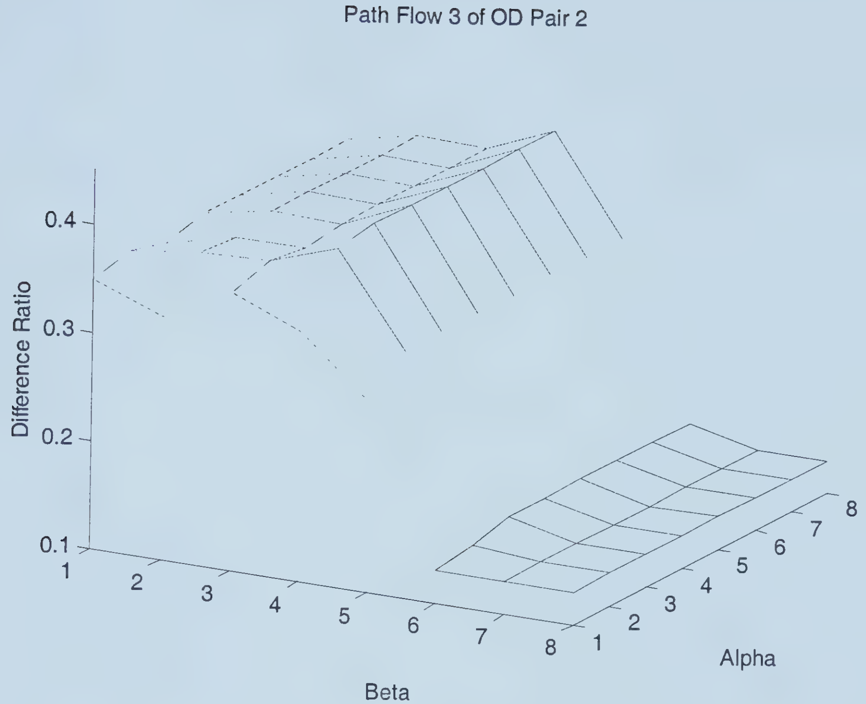


Figure 3-5: Path Flow 3 in OD Pair 2

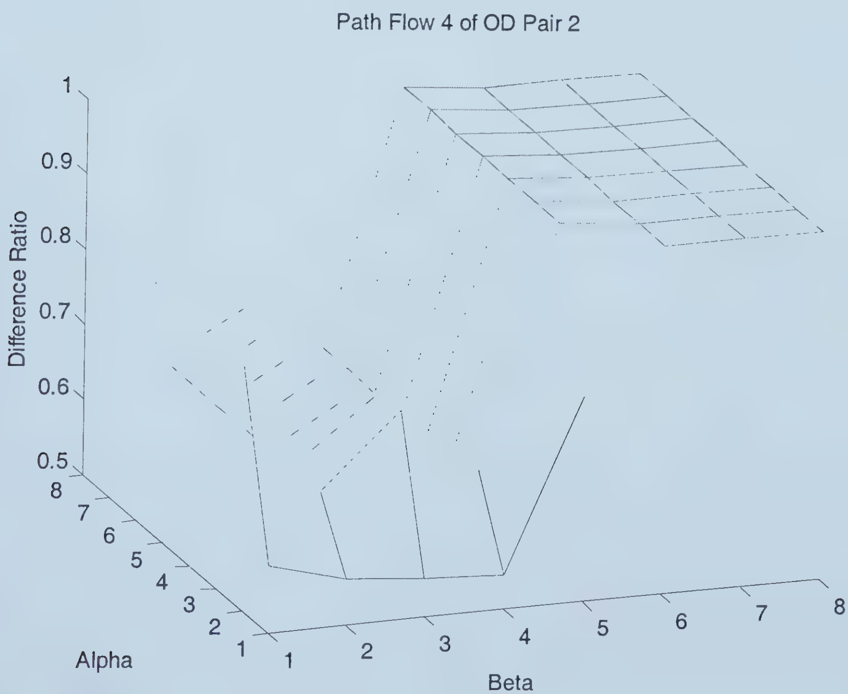


Figure 3-6: Path Flow 4 in OD Pair 2

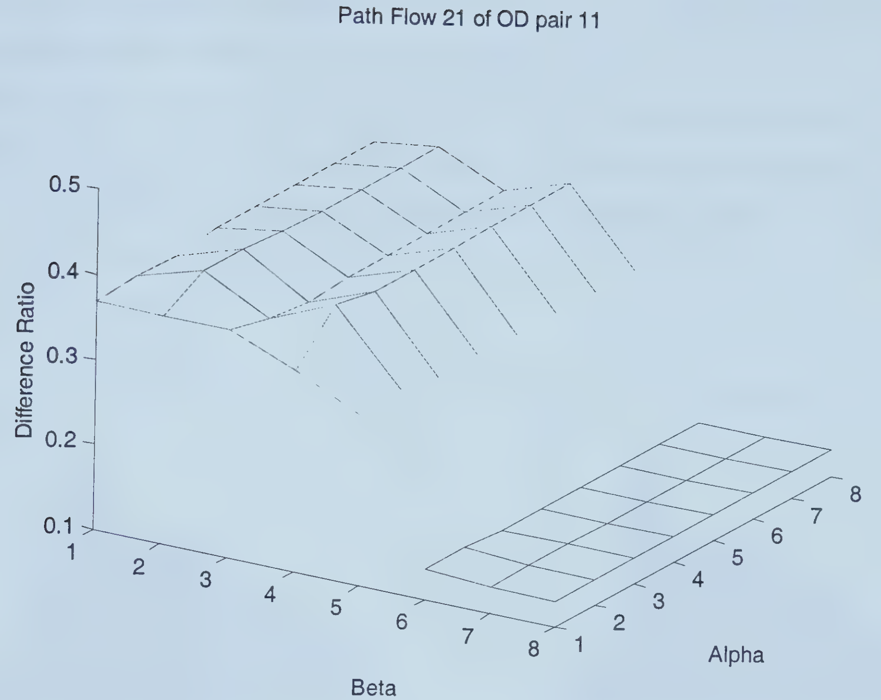


Figure 3-7: Path Flow 21 in OD Pair 11

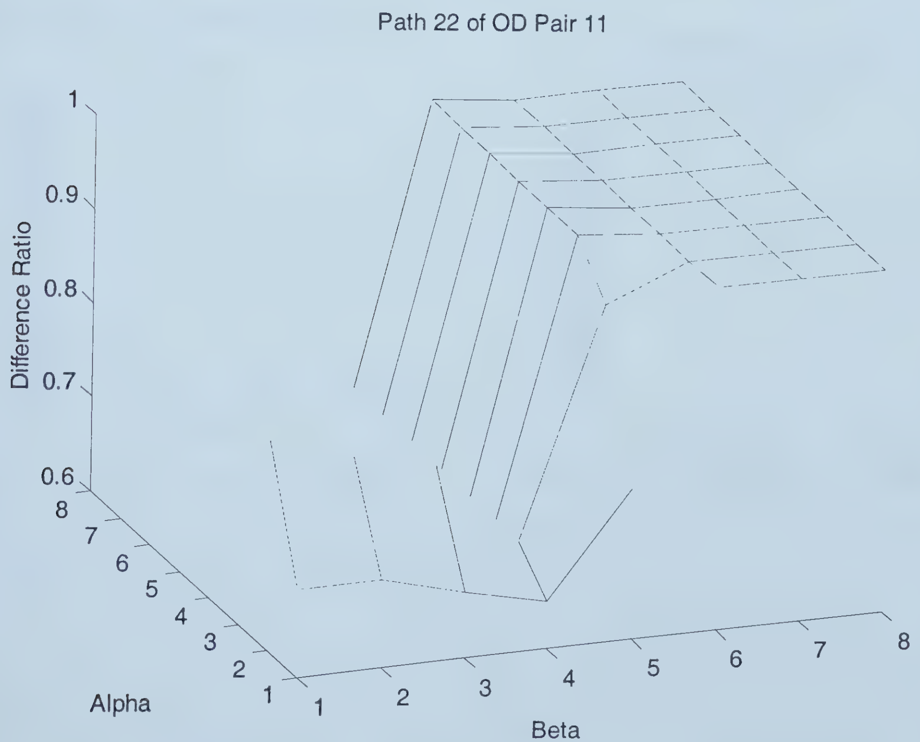


Figure 3-8: Path Flow 22 in OD Pair 11

The typical parameters in network are described in such a situation: the backbone path has the cheapest cost in 40, meanwhile the backup path has the higher cost in 80. We assume that they have same path capacities in 20 and compared to maximal external traffic flow in 10. Under these conditions, for the backbone and backup paths in all three OD pairs, the PDF in optimal flow solutions are graphically shown in Figures 3-9,10,11,12,13 and 14.

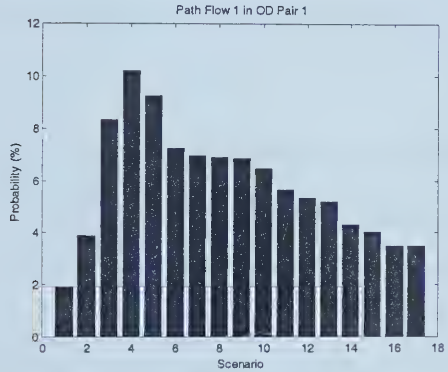


Figure 3-9: PDF, Path Flow 1 in OD Pair 1

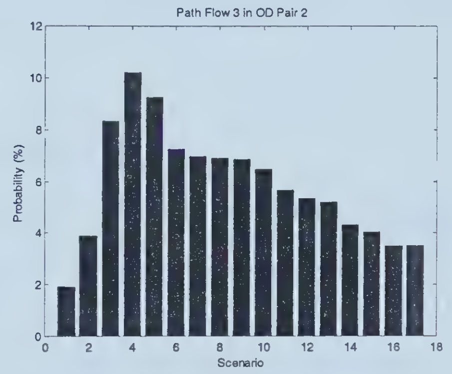


Figure 3-11: PDF, Path Flow 3 in OD Pair 2

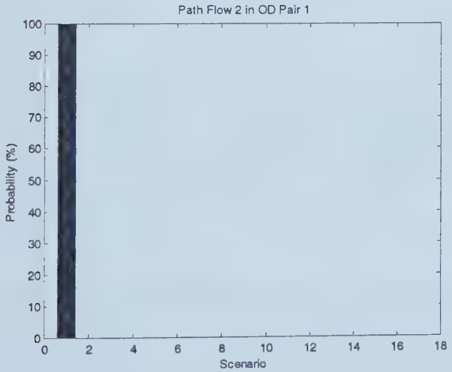


Figure 3-10: PDF, Path Flow 2 in OD Pair 1

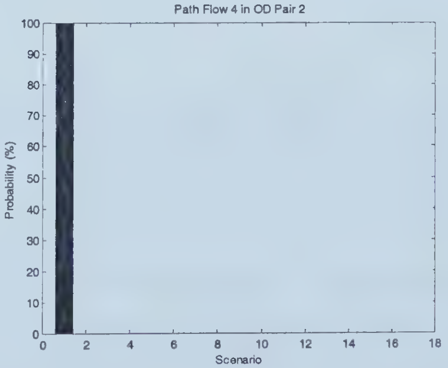


Figure 3-12: PDF, Path Flow 4 in OD Pair 2

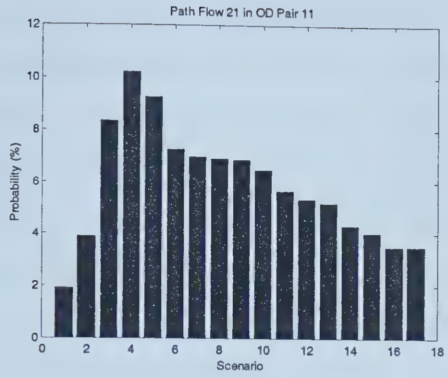


Figure 3-13: PDF, Path Flow 21 in OD Pair 11

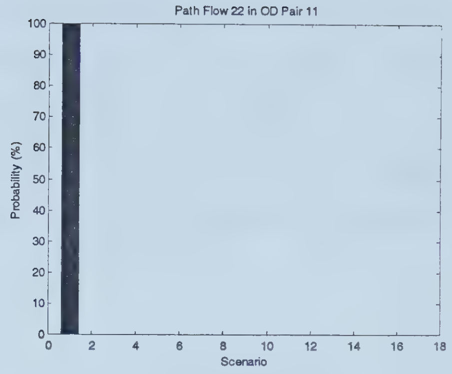


Figure 3-14: PDF, Path Flow 22 in OD Pair 11

3.1.5 Trend of Changing α (in Path Capacities)

As we discussed in previous section, *Difference Ratio* (DR) is designed to evaluate whether optimal path flow is sensitive to the selection of given constraint, such as α.

For the various realizations of α in simulation i for path1 of OD pair 1, based on the same results in Table 3-6, the DRs are summarized by their statistical moments and listed in Table 3-12.

Table 3-12: Path 1 in OD Pair 1

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
Average	0.2393	0.2813	0.2801	0.2812	0.2812	0.2812	0.2812	0.2812
Standard Deviation	0.0588	0.1135	0.1109	0.1122	0.1122	0.1122	0.1122	0.1122

Obviously, with eight sets of realization of α, β_1 has the smallest variance and average, whereas other seven sets have similar variances and averages. By observing Figure 3-3 (Trend of difference ratio of path 1), all eight sets of β_1 have similar trends. β_1 is therefore considered to be the most inactive with respect to the various realizations of α. Similarly, compared to other seven sets of β, this path (under the condition of β_1) has more similar PDF as that of external traffic flow, and shares more traffic than other paths.

Just as the previous statement on the property of performance function (objective function), the performance will be increased whenever the sets of flow approach the

capacity of their channels. Particularly, in our case about β_1, when the path traffic approaches the upper boundary of capacity (with lowest cost), in an optimal manner, other paths have to handle more traffic, even though they are with higher cost. In the previous simulation of small network, although path 2 is obviously shortest (or cheapest) path if compared to path 1, path 1 actually carried more traffic when there is not enough capacity in path 2.

In similar way, the DRs of other backbone paths are examined by using the same approach, and the summarized results are listed in the below Tables 3-13 and 3-14 respectively. The similar situation can be found.

Table 3-13: Path 3 in OD Pair 2

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
Average	0.2542	0.2740	0.2735	0.2762	0.2762	0.2762	0.2762	0.2762
Standard Deviation	0.0772	0.0997	0.0954	0.0995	0.0995	0.0995	0.0995	0.0995

Table 3-14: Path 21 in OD Pair 11

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
Average	0.2678	0.2778	0.2735	0.2749	0.2749	0.2749	0.2749	0.2749
Standard Deviation	0.0912	0.1061	0.1044	0.1058	0.1058	0.1058	0.1058	0.1058

As we discussed above, when path traffic approaches the upper boundary of path capacity (with lowest cost), other paths have to handle more traffic even though they are with higher cost. This is a particular situation under the condition of β_1. In this case study, two paths are assigned for each OD pair. The first one is the backbone path in shortest distance (or say lowest cost) to carry most of traffic, and the latter is the backup path in longest distance (or say highest cost) to route the extra traffic in case of a large amount of traffic arrivals. Thus, when the backbone traffic approaches the upper boundary of path capacity, in an optimal solution, the traffic flow of backup path has to be increased simultaneously.

By using the same procedure to evaluate the statistical moments of DR, we are going to examine whether relevant principles are applied to all backup path flows. For the traffic

flows in other backup paths, the DR results are summarized and listed in the Tables 3-15,16 and17 respectively.

Table 3-15: Path 2 in OD Pair 1

α	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
Average	0.7421	0.8526	0.8637	0.8629	0.8629	0.8629	0.8629	0.8629
Standard Deviation	0.2704	0.1135	0.0984	0.1022	0.1022	0.1022	0.1022	0.1022

Table 3-16: Path 4 in OD Pair 2

α	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
Average	0.7405	0.8447	0.8433	0.8363	0.8363	0.8363	0.8363	0.8363
Standard Deviation	0.2862	0.1470	0.1492	0.1664	0.1664	0.1664	0.1664	0.1664

Table 3-17: Path 22 in OD Pair 11

α	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
Average	0.7972	0.8656	0.8656	0.8634	0.8634	0.8634	0.8634	0.8634
Standard Deviation	0.1470	0.0815	0.1036	0.1114	0.1114	0.1114	0.1114	0.1114

Obviously, with eight sets of realizations of α, β_1 has the highest variance among eight sets of realizations of β, whereas other seven sets have similar variances and averages. Furthermore, by observing Figures 3-4,6 and 8 (Trend of Difference ratio for path 2,4 and 22), all eight sets of β still have similar trends. β_1 is therefore considered to be the most active with respect to the various realizations of α. In similar way, compared to the other seven sets of β, this path (under the condition of β_1) has less similar PDF than that of external traffic flow, and shares less traffic than other paths. This result exactly matches our conclusion mentioned in former section.

3.1.6 Trend of Changing β (in Costs)

According to our SA and WS procedure, it is also necessary to compare the characterizations of sensitivity in the various realizations of both α (capacity) and β (cost). The pervious evaluation approach for α can be modified and then is applied to the proposal on cost confections. For example, for the various realizations of α & β in path1 of OD pair 1, based on the same results in Table 3-6, the DRs are summarized by their statistical moments and listed in Table 3-18.

Table 3-18: Path 1 in OD Pair 1

	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8
Average	0.3626	0.3757	0.3808	0.3814	0.3142	0.1297	0.1307	0.1315
Standard Deviation	0.0040	0.0053	0.0052	0.0042	0.0018	0.0001	0.0000	0.0000
S.D./ Average	1.1%	1.4%	1.4%	1.1%	0.6%	0.0%	0.0%	0.0%

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
Average	0.2393	0.2813	0.2801	0.2812	0.2812	0.2812	0.2812	0.2812
Standard Deviation	0.0588	0.1134	0.1109	0.1121	0.1121	0.1121	0.1121	0.1121
S.D./ Average	24.6%	40.3%	39.6%	39.9%	39.9%	39.9%	39.9%	39.9%

A special ratio (*Standard Deviation/ Average)* is introduced to evaluate the most active factor between α and β because averages and standard deviations are not enough to support our analysis. In this situation, the percentage of value on the introduced measurement is a better solution to aid in looking at the statistical moment of DRs. Obviously, based on the statistical moments from above tables, there is the most active changing in β over α. Similarly, the DRs on other (optimal) path flows are examined by using the same approach, and listed in the below Tables 3-19,20,21,22 and 23 respectively.

Table 3-19: Path 2 in OD Pair 1

	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8
Average	0.7883	0.7565	0.7731	0.6300	0.9210	0.9680	0.9680	0.9680
Standard Deviation	0.0531	0.0432	0.0526	0.0024	0.0340	0.0000	0.0000	0.0000
S.D./ Average	6.73%	5.71%	6.80%	0.38%	3.69%	0.00%	0.00%	0.00%

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
Average	0.7421	0.8526	0.8637	0.8629	0.8629	0.8629	0.8629	0.8629
Standard Deviation	0.2704	0.1135	0.0984	0.1022	0.1022	0.1022	0.1022	0.1022
S.D./ Average	36.4%	13.3%	11.4%	11.8%	11.8%	11.8%	11.8%	11.8%

Table 3-20: Path 3 in OD Pair 2

	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8
Average	0.3554	0.3641	0.3677	0.3884	0.3031	0.1425	0.1307	0.1307
Standard Deviation	0.0001	0.0019	0.0001	0.0035	0.0008	0.0002	0.0000	0.0000
S.D./ Average	0.04%	0.52%	0.03%	0.91%	0.25%	0.16%	0.00%	0.00%

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
Average	0.2542	0.2740	0.2735	0.2762	0.2762	0.2762	0.2762	0.2762
Standard Deviation	0.0772	0.0997	0.0954	0.0995	0.0995	0.0995	0.0995	0.0995
S.D./ Average	30.4%	36.4%	34.9%	36.0%	36.0%	36.0%	36.0%	36.0%

Table 3-21: Path 4 in OD Pair 2

	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8
Average	0.7961	0.6343	0.6932	0.6307	0.9516	0.9680	0.9680	0.9680
Standard Deviation	0.0486	0.0064	0.0255	0.0088	0.0357	0.0000	0.0000	0.0000
S.D./ Average	6.11%	1.01%	3.67%	1.40%	3.75%	0.00%	0.00%	0.00%

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
Average	0.7405	0.8447	0.8433	0.8363	0.8363	0.8363	0.8363	0.8363
Standard Deviation	0.2862	0.1470	0.1492	0.1664	0.1664	0.1664	0.1664	0.1664
S.D./ Average	38.6%	17.4%	17.7%	19.9%	19.9%	19.9%	19.9%	19.9%

Table 3-22: Path 21 in OD Pair 11

	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8
Average	0.3768	0.3901	0.3577	0.3775	0.2980	0.1322	0.1307	0.1307
Standard Deviation	0.0002	0.0006	0.0002	0.0019	0.0001	0.0001	0.0000	0.0000
S.D./ Average	0.05%	0.15%	0.04%	0.50%	0.04%	0.04%	0.00%	0.00%

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
Average	0.7405	0.8447	0.8433	0.8363	0.8363	0.8363	0.8363	0.8363
Standard Deviation	0.2862	0.1470	0.1492	0.1664	0.1664	0.1664	0.1664	0.1664
S.D./ Average	38.6%	17.4%	17.7%	19.9%	19.9%	19.9%	19.9%	19.9%

Table 3-23: Path 22 in OD Pair 12

	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8
Average	0.8093	0.7732	0.7666	0.6479	0.9442	0.9680	0.9680	0.9680
Standard Deviation	0.0150	0.0072	0.0104	0.0020	0.0398	0.0000	0.0000	0.0000
S.D./ Average	1.85%	0.94%	1.36%	0.31%	4.21%	0.00%	0.00%	0.00%

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
Average	0.7972	0.8656	0.8656	0.8634	0.8634	0.8634	0.8634	0.8634
Standard Deviation	0.1470	0.0815	0.1036	0.1114	0.1114	0.1114	0.1114	0.1114
S.D./ Average	18.4%	9.4%	12.0%	12.9%	12.9%	12.9%	12.9%	12.9%

Based on the statistical moments from above tables, there are the most active changing in β over α. The result on our optimization problems exhibits the relative small ratio of the variance of path capacities to the variance of path costs. This implies that optimal path flow is sensitive to the selection of the given design for cost coefficients instead of cost capacities.

3.1.7 Summary

As the previous discussion on the property of performance function (objective function), the performance will be increased whenever the sets of flow approach the capacity of their channels. In our simulation case, it can be observed exactly when path traffic approaches the upper boundary of path capacity (with smaller cost). In an optimal manner, other paths are then assigned to handle more traffic even though they are with higher cost.

Furthermore, for the various realizations of both α (capacity constraint) and β (cost coefficients), sensitivity analysis was conducted to estimate the model characterization of sensitivity. The results on our optimization problem exhibit the relative small ratio of the variance of path capacities to the variance of path costs. This implies that optimal path flow is sensitive to the selection of the given design for cost coefficients over cost capacities.

3.2 Case study 2

Traffic pattern adopted in the last chapter is based on the Poisson assumption. In this chapter, we continue our effort to apply the WS & SA approaches to other traffic patterns. Consider following case,

3.2.1 Traffic Arrival Model in Simulation Procedure

The traffic demands have sixty-four scenarios and randomly change among 0, 5, 10 and 15. Each scenario has a probability, specified in Table 3-24. The graphical representation of PDF is shown in Figure 3-15.

Table 3-24: Scenarios and Probabilities

Scenario	1	2	3	4
Traffic rate	0	5	10	15
Probability	37.5%	37.5%	18.75%	6.25%

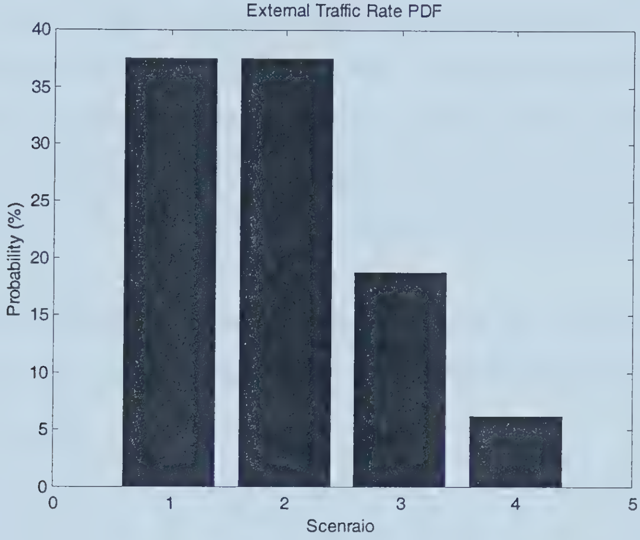


Figure 3-15: PDF of Traffic Rate

For each OD pair, sixty-four scenarios are arranged in totally different (or random) sequences. The behavior of external traffic requirement at OD pair 1 is also graphically shown in Figure 31. External traffic into other OD pairs has same PDF in OD pair 1.

3.2.2 Sensitivity Analysis in Flow Assignment Model

Based on similar considerations, our sensitivity analysis procedure is developed to investigate the ranges on changing both objective function coefficients (path cost or link cost) and constraint values (path capacity) over which the values of decisions variables in an optimal solutions will remain unchanged. Some detailed considerations on the variation of path capacity and cost will be introduced in following sections. Furthermore, as our discussion on WS and its considerations on the PDF of optimal solutions, a measurement of variance on optimal solutions, particularly on their statistical characterizations, is designed and described in later part of this chapter. In general, based

on the combined approaches of SA and WS, we employ the same measurement model to investigate overall characterizations of Flow (Routing) Assignment problem.

Capacity

In this SA procedure, we still concern the similar situations about two sets of constraint: path capacity and external traffic. For example, over what kind of range of changing capacity constraint as well as external traffic, objective function (minimal cost in our case) or optimal solution in this model is reported to be invariable. In short, it will be about how optimal path flow is sensitive to the selection of given design.

For the implementation of WS and SA procedure, the particular (random variables) realizations on the relevant path capacities are also modified and shown in Table 3-25.

Table 3-25: Path Capacities

Realizations	1	2	3	4	5	6	7	8
Backbone capacity	15	20	25	30	35	40	45	50
Backup capacity	15	20	25	30	35	40	45	50
Maximum flows	15	15	15	15	15	15	15	15
α	2	2.67	3.33	4	4.67	5.33	6	6.67

Cost

As we mentioned in SA procedure, the considerations on cost coefficients also interest us. For example, over such a range of changing cost coefficients, objective function (minimal cost in our case) or optimal solution is reported to be invariable. Then, in this case, one of our concerns will be how optimal path flow is sensitive to the selection of given design.

For the implementation of WS and SA procedure, the particular (random variables) realizations on the relevant path cost coefficients are also modified and shown in Table 3-26.

Table 3-26: Path Costs

Realizations	1	2	3	4	5	6	7	8
Backbone cost	40	40	40	40	40	40	40	40
Backup cost	2.5	5	10	20	40	80	160	320
β	6.25%	12.5%	25%	50%	100%	200%	400%	800%

Similarly, external traffic requirements at all OD pairs follow exactly same PDF, but in totally different (or random) sequences. Another numerical optimization program was designed to carry out this WS and SA procedure.

OD Pair

In this case study, all backbone paths pass one link only. There are three different types of backup path. Some pass up to two links, some up to three links, and others up to four links. Thus, three types of example on OD pair are selected to investigate the optimal flow solutions between their backbone and backup paths. They are also listed in Table 3-27.

Table 3-27: Example OD Pairs

OD pair index	Backbone path index	Backup path index	Backup path pass links
1	1	2	4
2	3	4	3
11	21	22	2

3.2.3 Analysis Procedure

Similarly, based on the combined considerations on both WS technique in SP and SA technique in applied OR, our simulation procedure is developed.

In this simulation procedure, SA approach is implemented by using the various realizations on eight sets of α (for capacity) and eight sets of β (for cost) simultaneously and then conducting the variance measurements with respect to other considerations from WS.

Under the various realizations of relevant paths and capacity, the estimations of the PDF of optimal flow solutions are the most important processes during WS procedure. Meanwhile, with consideration on SA, the estimations of PDF are applied to the (8 x 8 = 64 points of) measurements on variance ratios among the optimal flow solutions in backbone or backup paths, and external traffic flow (64 scenarios).

Since the PDF of demand traffic is known, we are going to estimate the PDF of optimal path flows by following such a procedure. Optimal path flows are classified into individual group by their thresholds. The lower bound of thresholds is calculated based on the mean value of current scenario and pervious one. The upper bound is the mean value of current scenario and next one and so on. The thresholds are listed in Table 3-28. Probabilities among individual range are then easily obtained.

Table 3-28: Thresholds List

Thresholds Index	Thresholds Lower Bound	Thresholds Upper Bound	Relevant Probability	Relevant Traffic rate	Probability
1	0	2.5	P_1	0	37.5%
2	2.5	7.5	P_2	5	37.5%
3	7.5	12.5	P_3	10	18.8%
4	12.5	+8	P_4	15	6.3%

The same variance measurement, DR, is applied to investigate the trend of probability distribution changing under various realizations of (8 x 8=) 64 sets of cost and capacity, and is calculated using the below formula,

$$Difference\ ratio = \sqrt{(P_1 - 37.5\%)^2 + (P_2 - 37.5\%)^2 + (P_3 - 18.8\%)^2 + (P_4 - 6.3\%)^2}$$

DR is still used to express the degree of similarity between two measured PDFs. The lower value of DR refers to very similar situation between two measured PDFs. Lower DR implies similar PDFs between optimal flow solution and external traffic flow. One path with lower DR is assigned more traffic flow than other paths with higher DR in same OD pair. Thus, for three typical OD pairs, (8 x 8 =) 64 points of trend on DR will be obtained for each path. The original data is shown by 3-29,30,31,32,33 and 34, and their graphical representations in Figures 3-16,17,18,19,20 and 21.

Table 3-29: Path 1 in OD Pair 1

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
α_1	0.3743	0.3743	0.3344	0.0988	0	0	0	0
α_2	0.3671	0.4069	0.3542	0.0585	0	0	0	0
α_3	0.3872	0.4210	0.3344	0.0733	0	0	0	0
α_4	0.4245	0.4026	0.3117	0.0733	0	0	0	0
α_5	0.4279	0.4245	0.3179	0.0765	0	0	0	0
α_6	0.4279	0.4436	0.3179	0.0765	0	0	0	0
α_7	0.4279	0.4436	0.3179	0.0765	0	0	0	0
α_8	0.4279	0.4436	0.3179	0.0765	0	0	0	0

Table 3-30: Path 2 in OD Pair 1

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
α_1	0.2529	0.2529	0.2733	0.5154	0.7552	0.7552	0.7552	0.7552
α_2	0.2471	0.2529	0.2451	0.6128	0.7552	0.7552	0.7552	0.7552
α_3	0.2286	0.1754	0.2733	0.6128	0.7552	0.7552	0.7552	0.7552
α_4	0.2176	0.2131	0.2733	0.6128	0.7552	0.7552	0.7552	0.7552
α_5	0.2108	0.1989	0.2873	0.6328	0.7552	0.7552	0.7552	0.7552
α_6	0.2108	0.1809	0.2873	0.6328	0.7552	0.7552	0.7552	0.7552
α_7	0.2108	0.1809	0.2873	0.6328	0.7552	0.7552	0.7552	0.7552
α_8	0.2108	0.1809	0.2873	0.6328	0.7552	0.7552	0.7552	0.7552

Table 3-31: Path 3 in OD Pair 2

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
α_1	0.3109	0.3515	0.2001	0	0	0	0	0
α_2	0.4026	0.4075	0.1862	0	0	0	0	0
α_3	0.4222	0.4169	0.2296	0	0	0	0	0
α_4	0.4222	0.3891	0.2176	0	0	0	0	0
α_5	0.4245	0.4122	0.1726	0	0	0	0	0
α_6	0.4245	0.4122	0.2061	0	0	0	0	0
α_7	0.4245	0.4122	0.2061	0	0	0	0	0
α_8	0.4245	0.4122	0.2061	0	0	0	0	0

Table 3-32: Path 4 in OD Pair 2

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
α_1	0.5899	0.5604	0.5500	0.5447	0.7748	0.9680	0.9680	0.9680
α_2	0.8256	0.6482	0.7463	0.6568	0.9768	0.9680	0.9680	0.9680
α_3	0.8256	0.6550	0.7337	0.6512	0.9768	0.9680	0.9680	0.9680
α_4	0.8256	0.6422	0.7031	0.6386	0.9768	0.9680	0.9680	0.9680
α_5	0.8256	0.6422	0.7031	0.6386	0.9768	0.9680	0.9680	0.9680
α_6	0.8256	0.6422	0.7031	0.6386	0.9768	0.9680	0.9680	0.9680
α_7	0.8256	0.6422	0.7031	0.6386	0.9768	0.9680	0.9680	0.9680
α_8	0.8256	0.6422	0.7031	0.6386	0.9768	0.9680	0.9680	0.9680

Table 3-33: Path 21 in OD Pair 11

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
α_1	0.3086	0.3240	0.2471	0.1914	0.0000	0.0000	0.0000	0.0000
α_2	0.4587	0.3872	0.2940	0.1105	0.0000	0.0000	0.0000	0.0000
α_3	0.4419	0.4075	0.3387	0.1639	0.0000	0.0000	0.0000	0.0000
α_4	0.4245	0.3872	0.3387	0.1269	0.0000	0.0000	0.0000	0.0000
α_5	0.4245	0.3872	0.3387	0.1307	0.0000	0.0000	0.0000	0.0000
α_6	0.4044	0.4075	0.3179	0.1105	0.0000	0.0000	0.0000	0.0000
α_7	0.4044	0.4075	0.3179	0.1105	0.0000	0.0000	0.0000	0.0000
α_8	0.4044	0.4075	0.3179	0.1105	0.0000	0.0000	0.0000	0.0000

Table 3-34: Path 22 in OD Pair 11

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
α_1	0.2661	0.2873	0.3452	0.4598	0.7552	0.7552	0.7552	0.7552
α_2	0.2176	0.2733	0.2856	0.5345	0.7552	0.7552	0.7552	0.7552
α_3	0.2519	0.2795	0.2642	0.4587	0.7552	0.7552	0.7552	0.7552
α_4	0.2661	0.2733	0.2642	0.4966	0.7552	0.7552	0.7552	0.7552
α_5	0.2661	0.2733	0.2642	0.5159	0.7552	0.7552	0.7552	0.7552
α_6	0.2586	0.2795	0.2890	0.4966	0.7552	0.7552	0.7552	0.7552
α_7	0.2586	0.2795	0.2890	0.4966	0.7552	0.7552	0.7552	0.7552
α_8	0.2586	0.2795	0.2890	0.4966	0.7552	0.7552	0.7552	0.7552

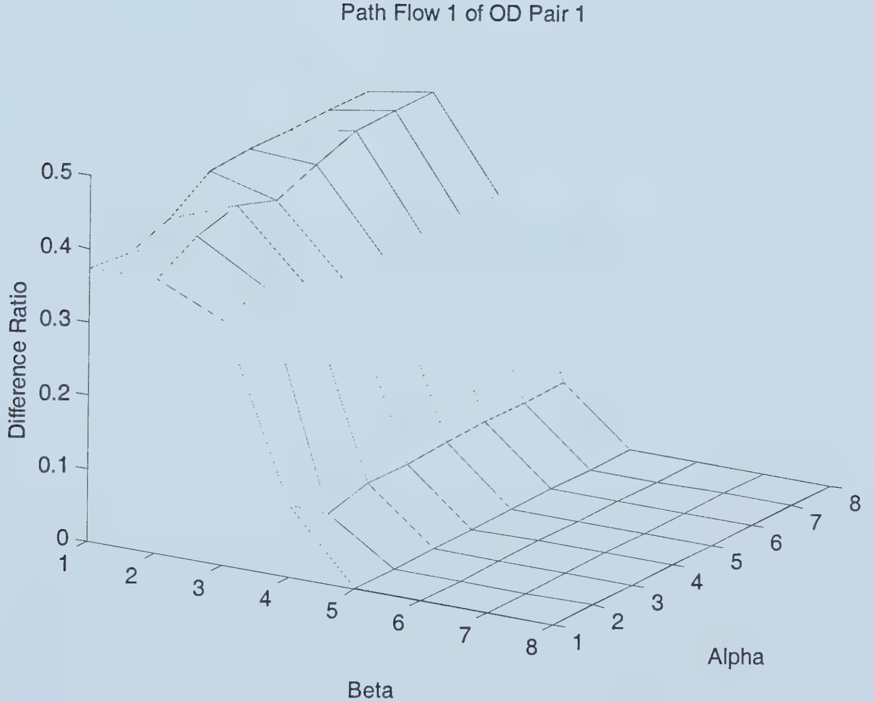


Figure 3-16: Path Flow 1 in OD Pair 1

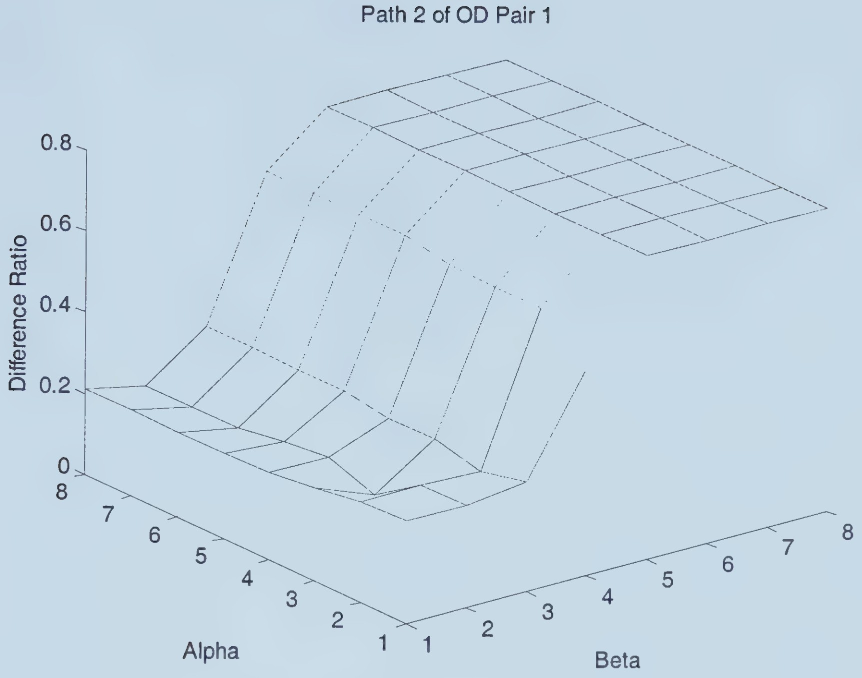


Figure 3-17: Path Flow 2 in OD Pair 1

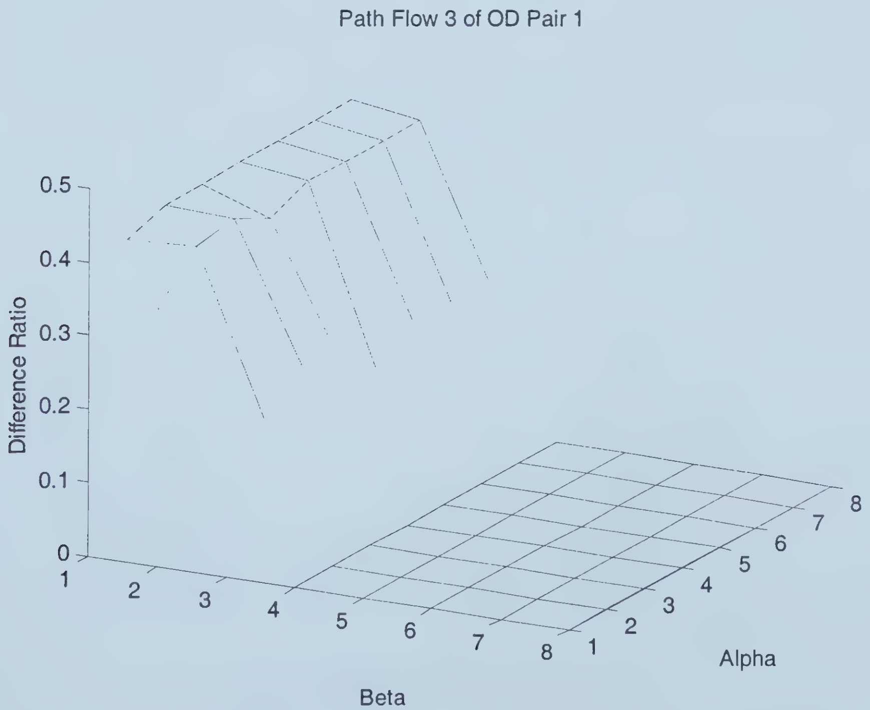


Figure 3-18: Path Flow 3 in OD Pair 2

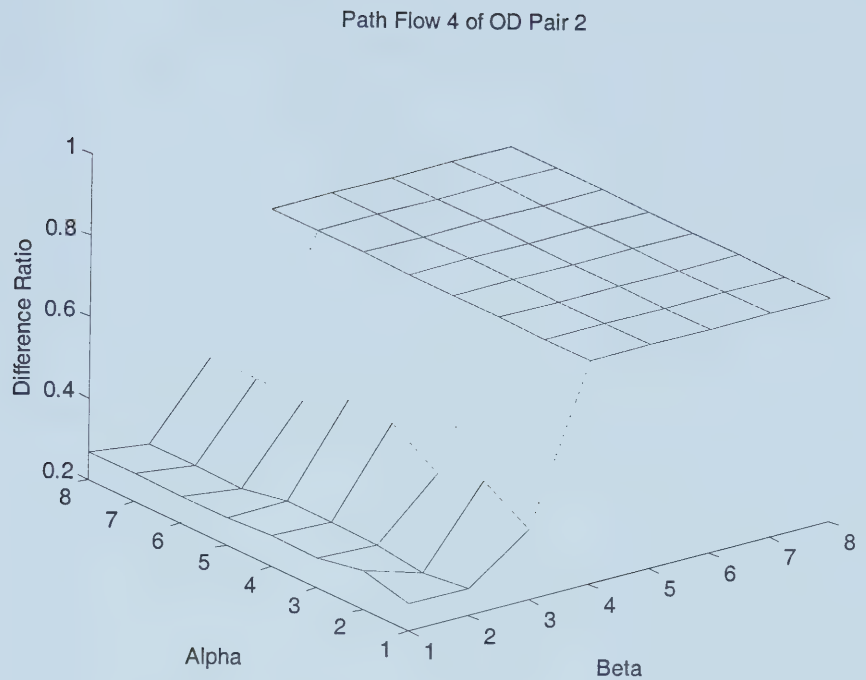


Figure 3-19: Path Flow 4 in OD Pair 2

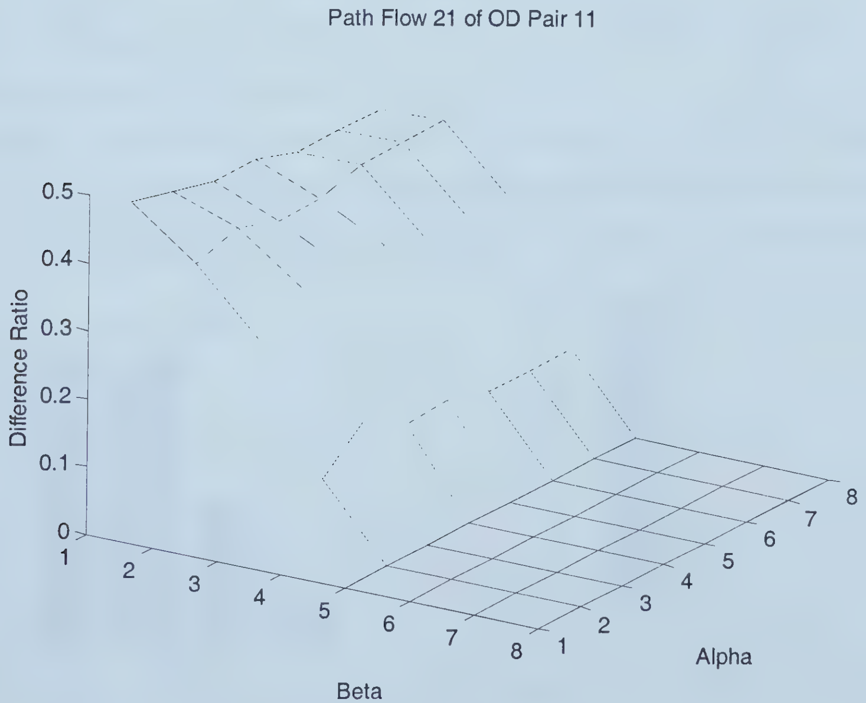


Figure 3-20: Path Flow 21 in OD Pair 11

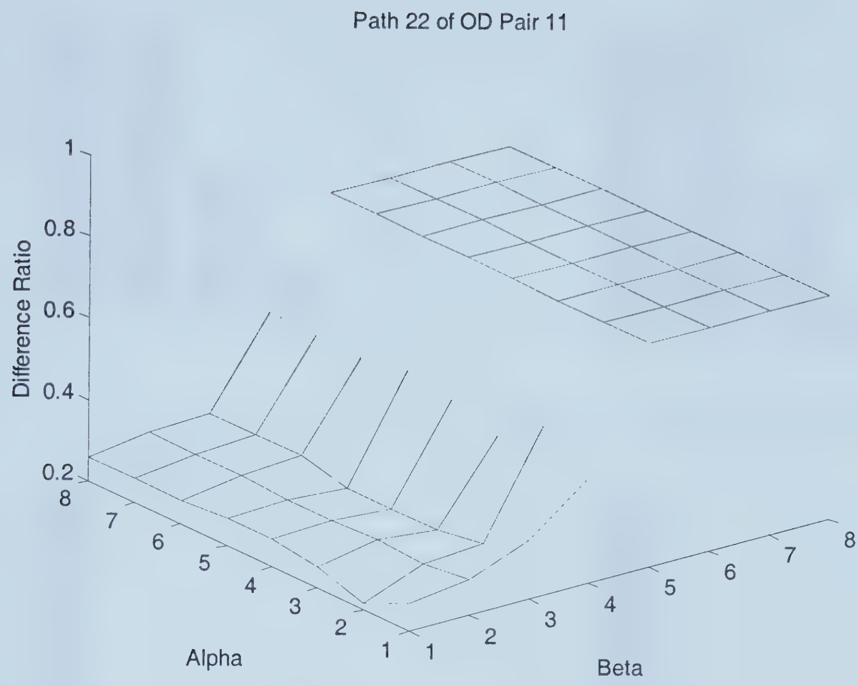


Figure 3-21: Path Flow 22 in OD Pair 11

The typical parameters in network are described in such a situation: the backbone path has the cheapest cost in 40, meanwhile backup path has the higher cost in 80. We assume that they have same path capacities in 30 and are compared to maximal external traffic flow in 15. Under these conditions, for the backbone and backup paths in all three OD pairs, the PDFs in optimal flow solutions are shown in Figures 3-22,23,24,25,26 and 27.

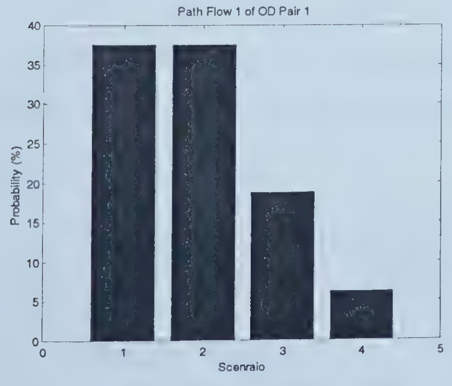


Figure 3-22: PDF, Path Flow 1 in OD Pair 1

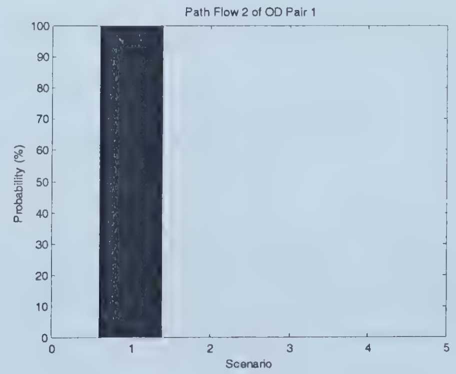


Figure 3-23: PDF, Path Flow 2 in OD Pair 1

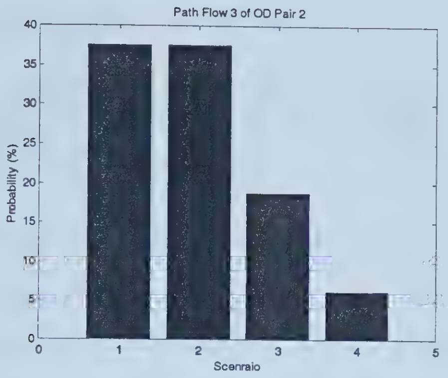


Figure 3-24: PDF, Path Flow 3 in OD Pair 2

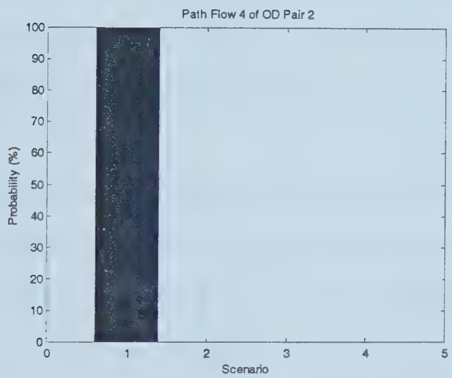


Figure 3-25: PDF, Path Flow 4 in OD Pair 2

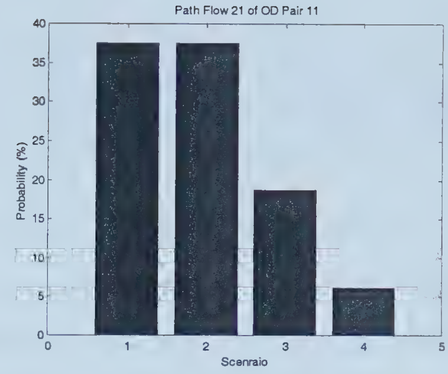


Figure 3-26: PDF, Path Flow 21 in OD Pair 11

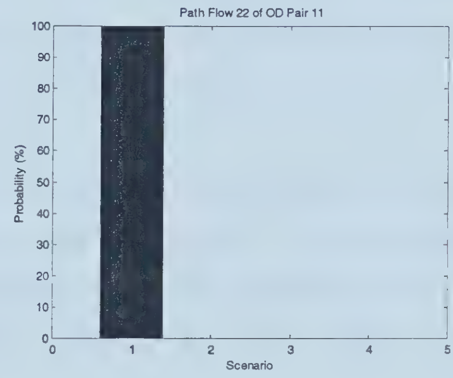


Figure 3-27: PDF, Path Flow 22 in OD Pair 11

3.2.4 Trend of Changing α (in Path Capacities)

As we discussed in previous chapter, DR is used to evaluate whether optimal path flow is sensitive to the selection of given constraint, such as α. In case study 2, for various realizations of α for all backbone paths and based on the simulation results, the DRs are summarized by their statistical moments and listed in Tables 3-35,36 and 37.

Table 3-35: Path 1 in OD Pair 1

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
Average	0.1477	0.1483	0.1520	0.1515	0.1559	0.1582	0.1582	0.1582
Standard Deviation	0.2272	0.2532	0.2596	0.2612	0.2759	0.2865	0.2865	0.2865

Table 3-36: Path 3 in OD Pair 2

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
Average	0.1477	0.1483	0.1520	0.1515	0.1559	0.1582	0.1582	0.1582
Standard Deviation	0.2272	0.2532	0.2596	0.2612	0.2759	0.2865	0.2865	0.2865

Table 3-37: Path 5 in OD Pair 11

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
Average	0.1339	0.1563	0.1690	0.1597	0.1601	0.1550	0.1550	0.1550
Standard Deviation	0.1545	0.2635	0.2744	0.2570	0.2568	0.2506	0.2506	0.2506

Similar results are found again in case study 2. With eight sets of realizations of α, β_1 has the smallest variance and average among eight sets of realizations of β, whereas other seven sets of β have similar variances and averages. Furthermore, by observing Figures 3-16,18 and 20, all eight sets of β have similar trends. Therefore, β_1 is considered to be the most inactive with respect to the various realizations of α. If compared to the other seven sets of β, this path's (under the condition of β_1) optimal flow has the most similar PDF as that of external traffic flow. This path share more traffic than other paths.

Just as our previous statement on the property of performance function, the performance will be increased whenever the sets of flow approach the capacity of their channels. Particularly, in our case, when path traffic approaches the upper boundary of path capacity (with smaller cost), in an optimal manner, other paths have to handle more traffic even though they are with higher cost.

Furthermore, the DRs on other backbone paths are examined by using the same approach. The results are listed in Tables 3-38,39 and 40.

Table 3-38: Path 2 in OD Pair 1

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
Average	0.5394	0.5473	0.5389	0.5422	0.5438	0.5416	0.5416	0.5416
Standard Deviation	0.6218	0.4453	0.4916	0.4724	0.4823	0.4950	0.4950	0.4950

Table 3-39: Path 4 in OD Pair 2

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
Average	0.7405	0.8447	0.8433	0.8363	0.8363	0.8363	0.8363	0.8363
Standard Deviation	0.2862	0.1470	0.1492	0.1664	0.1664	0.1664	0.1664	0.1664

Table 3-40: Path 22 in OD Pair 11

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
Average	0.5474	0.5415	0.5344	0.5401	0.5425	0.5431	0.5431	0.5431
Standard Deviation	0.5681	0.4250	0.4185	0.4093	0.4079	0.3971	0.3971	0.3971

Obviously, with eight sets of realizations of α, β_1 has the highest variance among eight sets of β, whereas the other seven sets have similar variances and averages. By observing Figures 3-17,19 and 21 (Trend of Difference Ratio for Path 2,4 and 12), all eight sets of β have similar trends. Therefore, β_1 is considered to be the most active with respect to the various realizations of α. If compared to the other seven sets of β, this path (under the condition of β_1) has less similar PDF as that of external traffic flow, and shares less traffic than other paths. This result exactly matches our statements mentioned in the former chapter.

3.2.5 Trend of Changing β (in Costs)

Again, according to our SA procedure, it is necessary to compare the characterizations of sensitivity in various realizations of α (capacity) as well as β (cost). The special ratio (*Standard Deviation/ Average)* is still applied to evaluate the most active factor between α and β. In case study 2, for various realizations of β in all paths, the original and their DRs are also summarized by their statistical moments and listed in Tables 3-41,42,43,44,45 and 46.

Table 3-41: Path 1 in OD Pair 1

	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8
Average	0.4081	0.4200	0.3258	0.0762	0.0000	0.0000	0.0000	0.0000
Standard Deviation	0.0051	0.0043	0.0014	0.0008	0.0000	0.0000	0.0000	0.0000
S.D./ Average	1.25%	1.01%	0.43%	1.10%	N/A	N/A	N/A	N/A
	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
Average	0.1477	0.1483	0.1520	0.1515	0.1559	0.1582	0.1582	0.1582
Standard Deviation	0.2272	0.2532	0.2596	0.2612	0.2759	0.2865	0.2865	0.2865
S.D./ Average	153.8%	170.7%	170.8%	172.4%	177.0%	181.0%	181.0%	181.0%

Table 3-42: Path 2 in OD Pair 1

	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8
Average	0.2237	0.2045	0.2768	0.6106	0.7552	0.7552	0.7552	0.7552
Standard Deviation	0.0021	0.0073	0.0015	0.0110	0.0000	0.0000	0.0000	0.0000
S.D./ Average	0.95%	3.58%	0.54%	1.81%	N/A	N/A	N/A	N/A
	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
Average	0.5394	0.5473	0.5389	0.5422	0.5438	0.5416	0.5416	0.5416
Standard Deviation	0.6218	0.4453	0.4916	0.4724	0.4823	0.4950	0.4950	0.4950
S.D./ Average	116.1%	81.4%	91.2%	87.1%	88.7%	91.4%	91.4%	91.4%

Table 3-43: Path 3 in OD Pair 2

	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8
Average	0.4070	0.4017	0.2031	0.0000	0.0000	0.0000	0.0000	0.0000
Standard Deviation	0.0109	0.0034	0.0022	0.0000	0.0000	0.0000	0.0000	0.0000
S.D./ Average	2.69%	0.84%	1.07%	N/A	N/A	N/A	N/A	N/A
	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
Average	0.1078	0.1245	0.1336	0.1286	0.1262	0.1303	0.1303	0.1303
Standard Deviation	0.1673	0.2387	0.2620	0.2446	0.2526	0.2567	0.2567	0.2567
S.D./ Average	155.1%	191.7%	196.1%	190.2%	200.2%	196.9%	196.9%	196.9%

Table 3-44: Path 4 in OD Pair 2

	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8
Average	0.2725	0.2583	0.4317	0.7552	0.7552	0.7552	0.7552	0.7552
Standard Deviation	0.0008	0.0008	0.0084	0.0000	0.0000	0.0000	0.0000	0.0000
S.D./ Average	0.28%	0.31%	1.94%	N/A	N/A	N/A	N/A	N/A

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
Average	0.5857	0.5959	0.5909	0.5997	0.5986	0.5892	0.5892	0.5892
Standard Deviation	0.3906	0.3579	0.3721	0.3505	0.3565	0.3864	0.3864	0.3864
S.D./ Average	66.7%	60.1%	63.0%	58.4%	59.5%	65.6%	65.6%	65.6%

Table 3-45: Path 21 in OD Pair 11

	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8
Average	0.4089	0.3894	0.3139	0.1319	0.0000	0.0000	0.0000	0.0000
Standard Deviation	0.0142	0.0056	0.0068	0.0064	0.0000	0.0000	0.0000	0.0000
S.D./ Average	3.47%	1.44%	2.16%	4.87%	N/A	N/A	N/A	N/A

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
Average	0.1339	0.1563	0.1690	0.1597	0.1601	0.1550	0.1550	0.1550
Standard Deviation	0.1545	0.2636	0.2744	0.2570	0.2568	0.2506	0.2506	0.2506
S.D./ Average	115.4%	168.6%	162.4%	161.0%	160.3%	161.6%	161.6%	161.6%

Table 3-46: Path 22 in OD Pair 11

	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8
Average	0.2555	0.2782	0.2863	0.4944	0.7552	0.7552	0.7552	0.7552
Standard Deviation	0.0018	0.0002	0.0049	0.0046	0.0000	0.0000	0.0000	0.0000
S.D./ Average	0.71%	0.06%	1.73%	0.92%	N/A	N/A	N/A	N/A

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
Average	0.5474	0.5415	0.5344	0.5401	0.5425	0.5431	0.5431	0.5431
Standard Deviation	0.5681	0.4250	0.4185	0.4093	0.4079	0.3971	0.3971	0.3971
S.D./ Average	103.8%	78.5%	78.3%	75.8%	75.2%	73.1%	73.1%	73.1%

Obviously, based on the statistical moments from above tables, there is the most active changing in β over α.

3.2.6 Summary

Both case studies 1 & 2 can be considered to match the property of performance function, the performance will be increased whenever the sets of flow approach their upper boundary for these flows defined by the capacity constraints. In both cases, when backbone path traffic approaches the upper boundary of path capacity (with smaller cost), in an optimal manner, other (backup) paths are then assigned to handle more traffic even though they are with higher cost.

Furthermore, for various realizations of both α (capacity constraint) and β (cost coefficients), SA and WS approaches were conducted to estimate the model characterization of sensitivity. The results from case study 1 and 2 exhibit the relative small ratio of the variance of path capacities to the variance of path costs. This implies that optimal path flow is sensitive to the selection of the given design for cost coefficients over cost capacities.

Chapter 4 Conclusions

Modern communication paradigms are turning to B-ISDN protocols based on MC flow approaches.

Broadband-ISDN was developed to support integrated broadband services like high-speed-data service, videophone, video conferencing, CATV services as well as traditional ISDN services such as phone and telex. Various services are with transmission and switch speeds from 155 Mbps, 622 Mbps and up to 2.4 Gbps. One of the representative realizations in B-ISDN is the ATM technology. ATM technology is a connection-oriented service in a packet switched network. Before any communication is set up, the end- to-end connection is established and named a virtual channel connection. Then, the performance of network will be improved, the connection sharing common paths are grouped into a single unit, called virtual path.

Interest in optimal design of B-ISDNs has been high. For example, in B-ISDN/ATM, a careful selection of VPs may result in a VP layout that provides a reliable scheme of VCs planing with low connection and switching cost, so that a network is able to successfully deal with unexpected traffic loads. A significant amount of results has been published since 1990. Most proposed approaches were developed in two disciplines of mathematics: Graph Theory and Mathematical Programming.

In conventional network studies, the statistical characterizations of traffic arrivals are regularly considered without significantly change over time. As a result, network performance could be evaluated in terms of average values. However, as more and more new technologies are emerging, these conventional approaches seem without enough matching. In B-ISDN, various services are with transmission and switch speeds from 155 Mbps, 622 Mbps and up to 2.4 Gbps. Thus, several switching techniques, such as ATM, are developed and improved to manage the complicated situations in modern

communication network for switching signals ranging from 10s of bps to 100s of Mbps and service time distribution ranging from a few seconds to several hours

Therefore there is a need to introduce a new methodology to characterize the stochastic features of traffic behaviors in B-ISDNs. The modern traffic flow among communication network are either dynamic or stochastic, or both. In such a situation, the traditional approach using mean value of models is becoming invalid. Indeed, stochastic model is considered more suitable candidate to current practical circumstances. This thesis tries to integrate the SP methodology into MC network models. Mathematically, SP is an extension of MP to the stochastic space. Much attention is paid to optimization strategies adapted into the environment exemplified by stochastic traffic demands

One of the typical approaches adapted by SP is the WS method. The simple description of this method is to make a decision after all scenarios are observed. In network applications, the WS approach needs to be integrated with the stochastic effect of traffic arrivals in optimization network problem. This is one of our contributions in this thesis.

The Poisson model has been considered as a dominant role in modern communication development. Particularly, since the B-ISDN concept of commodity classes is relevant to traffic classes, the Poisson assumption on traffic is reasonable application. This is because, a path is typically an end-to-end connection and several types of traffic can be described by the Poisson pattern at the connection layer. Therefore, it is widely recognized with theoretical significance, and is helpfully applied to many analytical circumstances. For Poisson model, the PDF on arrival rate λ_n was developed as such a formula,

$$\rho(\lambda_n) = \lambda \cdot e^{-\lambda\left(\frac{1}{\lambda_n}\right)} \cdot \left| -\frac{1}{\lambda_n^2} \right| = \frac{\lambda}{\lambda_n^2} \cdot e^{-\frac{\lambda}{\lambda_n}}$$

Accordingly, this Poisson traffic model is integrated into WS and SA approaches and is applied to FA model described through several case studies for small and practical Unite States network.

Moreover, the traditional formulations of general models are considered as the link-node incidence. But, various types of traffic like CBR, VBR, UBR and ABR flows are transmitted over paths between connections of a set of OD pairs in practical B-ISDN network. Therefore, it is necessary to refer link flows to path flows and change the design variables respectively. Thus, the induced model becomes the path-node incidence.

Particularly, SA procedure is to investigate the ranges on changing both objective function coefficients (path cost) and constraint values (path capacity and external traffic) over which the values of decisions variables (in an optimal solutions) will remain unchanged. Furthermore, based on the considerations on both approaches of SA and WS, we develop a new measurement model to investigate overall characterizations of FA problem. This is also the main contribution throughout our research on B-ISDN optimization.

In this model, a metric called the DR is developed to characterize the changing trend of the PDF under various realizations of cost, capacity and external traffic. It expresses the degree of similarity between two measured PDFs. For example, lower DR refers to very similar situation between two measured PDFs. If this concept is applied to our FA problems, lower DR implies similar PDFs between optimal flow solution and external traffic flow. The path is assigned more traffic flow than other paths with higher DRs in same OD pair.

As the property of performance function in generic FA models, the performance will be increased whenever the sets of flow approach the capacity of their channels. In simulation cases, it can be observed exactly when path traffic approaches the upper boundary of path capacity (with smallest cost), in an optimal manner, other paths are then assigned to handle more traffic even though they are with higher cost.

Furthermore, with various realizations of both α (capacity constraint and external traffic) and β (cost coefficients), SA and WS approaches were employed to estimate the model

characterization of sensitivity in design variables. The results on optimization problem exhibit the relative small ratio of the variance of path capacities to path costs. This implies that optimal path flow is sensitive to the selection of the given design for cost coefficients over path capacities. Results are also reported in both tabular and graphical forms.

In future work, the similar SP & SA procedure can also be developed to CA problem and *Capacity and Flow Assignment* (CFA) problem.

Appendix 1: Probability Density Functions of Random Variable's Functions

Suppose X is a random variable, Y is a function of X. In symbols, we can write to indicate this function relationship as $Y=g(X)$, where g is continuous and real function. Under this condition, Y is also a random variable. In certain situation, it is necessary to calculate the probability density functions $Y=g(X)$ based on the known probability density functions of random variable X. In the following part of this section, we mainly deal with the continuous variable only.

Theorem.

Let a continuous random variable X has the probability density functions $f(x)$, and the function $y=g(x)$ is differentiable everywhere, *i.e.*, $g'(x) > 0$ (or $g'(x) < 0$) for any x. Thus, $Y=g(X)$ is also a continuous random variable, and its probability density functions is;

$$\psi(y) = \{ \begin{matrix} f[h(y)]\,|h'(y)|, \alpha < y < \beta \\ 0, others \end{matrix}$$

Where $h(y)$ is the inverse function of $g(x)$,

$\alpha = min\{g(-\infty), g(+\infty)\}$,

$\beta = max\{g(-\infty), g(+\infty)\}$.

Proof.

For any x, if there is always $g'(x) > 0$ (or $g'(x) < 0$), $g(x)$ is monotone increasing (or decreasing), therefore its inverse function $h(y)$ is existing, $h(y)$ is monotone increasing (or decreasing) and derivable within the range of (α, β).

1) Assume, $h(y)$ is single tone increasing, shown in diagram p1-1, the probability distribution function of Y is,

$$F_Y(y) = P\{Y \le y\} = P\{g(X) \le y\} = P\{X \le h(y)\} = \int_{-\infty}^{h(y)} f(x)dx$$

Thus, the probability density function is,

$$\psi(y) = F_Y'(y) = f[h(y)]h'(y), h'(y) > 0,$$
$$g(-\infty) < y < g(+\infty)$$

2) Assume, *h(y)* is monotone decreasing, the probability distribution function of *Y* is,

$$F_Y(y) = P\{Y \le y\} = P\{g(X) \le y\} = P\{X \le h(y)\} = \int_{h(y)}^{+\infty} f(x)dx$$

Thus, the probability density function is,

$$\psi(y) = F_Y'(y) = -f[h(y)]h'(y), h'(y) < 0,$$
$$g(+\infty) < y < g(-\infty)$$

To combine the results from 1 & 2, then get the probability density function about *Y=g(x)*.

Moreover, if *f(x)* is 0 when it is out of the rang [a,b], then *g(x) >0 (or g(x) <0)* exists only if it is inside the rang [a,b], also

α*=min{g(-∞),g(+∞)},*

β*=max{g(-∞),g(+∞)}.*

Appendix 2: Architecture of Software

The general formulation for a nonlinear optimization model can be expressed as follows:

Minimize

$$f(X)$$

Subject to

$$g(X) \geq 0$$

Where f: $R^n \rightarrow R^1$, g: $R^n \rightarrow R^m$. $f(x)$ is called the objective function and $g(X)$ the constraint function. The feasible region of X is represented by the region enclosed by $g(X)=0$.

Most effective methods for solving the optimization model above are known as indirect methods in that the constrained problem is transferred to an unconstrained one. The solving methodology consists of two levels:

- Convert the constrained formulation to an unconstrained one, and
- Minimize the unconstrained problem.

One of the widely applied methods used in level 1 is known as the exterior penalty function method. The major advantage of the exterior penalty function method is that a infeasible starting point X0 is allowed. It has, however, an inherent weakness. The value of the penalty factor, an integrated parameter, may become very large to force the optimization process to converge. To overcome this disadvantage of the exterior penalty function method, someone proposed the augmented Lagrangian multiplier method. Later, augmented Lagrangian multiplier method is applied to inequality constrained problems, and some general computational properties are established.

In augmented Lagrangian multiplier method, the augmented objective function is formulated as:

$$L(X,r,h) = f(x) + r\sum_{i=1}^{n} \| \min imum[0, g_i(X) + h_i / r] \|$$

where r and h_i are the penalty factor and the multiplier, receptively.

The augmented Lagrangian multiplier method shares with the exterior penalty function method the property that an infeasible starting points is allowed but does not suffer the problems the exterior penalty function method exhibits. As a relative new technique in optimization, augmented Lagrangian multiplier method has not yet been widely applied to solve practical problems. In our simulation software, augmented Lagrangian multiplier method is implemented to solve the optimization problem of network model.

At level 2 of the solving process, the conjugate direction method is implemented. The conjugate direction method has obtained a good reputation due to its robustness for solving complex nonlinear problems. It is expected to be appropriate for the range query optimization model because of the basic product formulation in the model.
The conjugate direction method requires searching the optimal points along several directions in the n-dimensional space. The parabolic interpolation method is chosen for carrying out this task as it exhibits good performance when incorporated in the conjugate direction method. The three major numerical algorithms described above as well as a few working subroutines are implemented to form a compact software package. Fortran is used as the implementation language since it is appropriate for scientific calculations.

The optimization software package consists of a main program and six subroutines: ALMM, CDM, PIM, XSTEP, RE and STOCH. Its size is about 600 lines of source code. Brief description follows:

STOCH: the subroutine incorporating stochastic parameters into the constructions of the objective function, the constraint function(s), and the argumented objective function.

XSTEP: the subroutine iterating the argument X and calling STOCH

RE: the subroutine swapping data. It is a working routine to simplify the program structure.

PIM: the subroutine executing the parabolic interpolation process and calling XSTEP and RE.

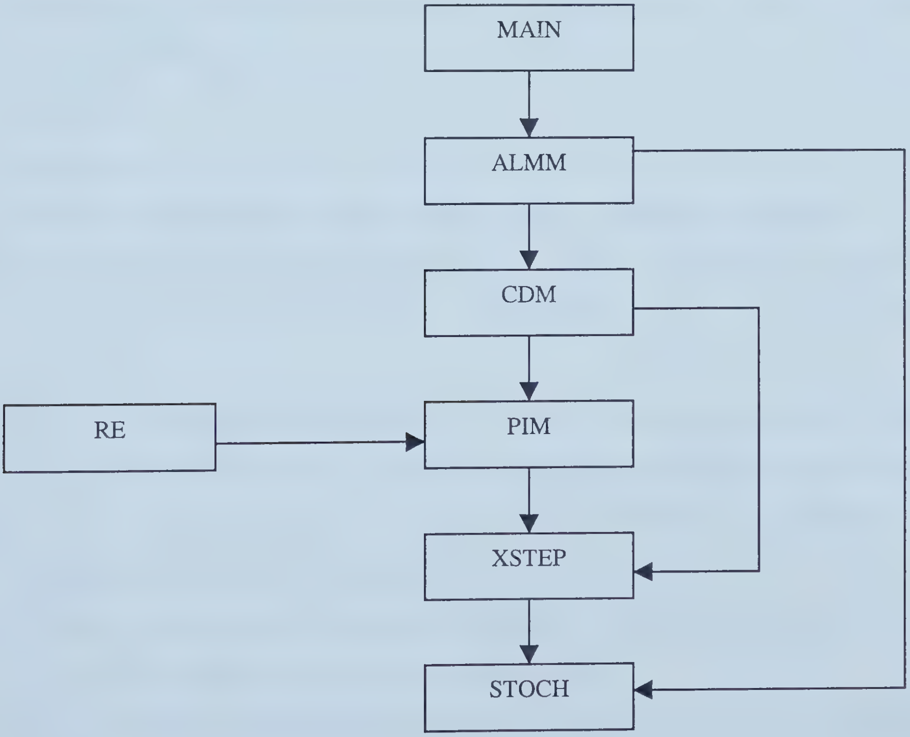
CDM: the subroutine performing the unconstrained minimization. Specifically, its function are:

- Call PIM to seek the optimal points along the initial set of conjugate directions;
- After searching n direction, construct the (n+1)th direction and use Powell's criterion to determine I fit is worthwhile choose this new direction to replace one of the previous directions;
- Check the convergence condition.

ALMM: the subroutine performing the augmented Lagrangian multiplier method. Specifically, it is functions are:

- Construct the initial set of conjugate directions;
- Call STOCH to calculate the objective function's value at the starting point to estimate the base value used for normalization;
- Call CDM
- Check the convergence condition

The structure of the software is shown in below.



Reference

[1] N. Anerousis and A. Lazar, "*Virtual path control for ATM networks with call level quality of service guarantees*", IEEE/ACM Trans. on Networking, 1998, 6(2), pp.222- 236.

[2] M. Bazaraa and J. Jarvis, "*Linear Programming and Network Flows*", Wiley, New York, 1977.

[3] D. Bertsekas and R. Gallager, "*Data Networks (2^{nd} ed.)*", Prentice Hall, NJ, 1992.

[4] S. P. Bradley and A.C. Hax, "*Applied mathematical programming*", Addison-Wesley, Massachusetts, 1977.

[5] Chlamtac, A. Farag'o, and T. Zhang, "*Generation of optimal transition count tests*", IEEE/ACM Trans. on Networking, 1994, 2(6), pp.581-587.

[6] M. Crovella and A. Bestavros, *"Self-similarity in World Wide Web traffic: evidence and possible causes*", IEEE/ACM Trans. on Networking, 1997, 5(6), pp.835-846.

[7] M. Dempster, "*Stochastic Programming*", Academic Press, New York, 1980.

[8] R. Dembo, "*Scenario optimization*", Annals of Operations Research, vol.30,1991, pp.63-80.

[9] M. Gerla and L. Kleinrock, "*On the topological design of distributed computer* networks", IEEE Trans. Communications, 1977; COM-25(1), pp.48-60.

[10] W. Leland, M. Taqqu, W. Willinger and D. Wilson, "*On the self-similar nature of ethernet traffic (Extended Version)*", IEEE/ACM Trans. on Networking, 1994, 2(1), pp.1-15.

[11] X. Liu, "A stochastic programming approach for range query retrieval problems", accepted by IEEE Trans. on Knowledge and Data Engineering, March 2001.

[12] X. Liu, "Network optimization with stochastic traffic flows", submitted to International Journal of Network Management, 2001.

[13] X. Liu, "A stochastic programming approach for optimization design of survivable networks", submitted to Electronics Letters, 2001.

[14] X. Liu, "The Role of Stochastic Programming in Network Design", submitted to Computer Networks, 2001.

[15] X. Liu, "A capacity assignment model with analytical solution", submitted to Electronics Letters, 2001.

[16] X. Liu, M. Mandal, and Y. Li, "On Mitigating Congestion for Stochastic Traffic Flows in Satellite ATM Networks", Submitted to IEEE/ACM Trans. on Networking, 2001.

[17] L. Kleinrock, "Queuing Systems: Volume II, Computer Applications", Wiley, New York, 1976.

[18] L. Kleinrock, "Queueing Systems: Volume I, Theory", Wiley, New York, 1975.

[19] W. Mendenhall and R. J. Beaver, "Introduction to Probability and Statistics", Duxbury, Belmont, 1994.

[20] P.S. Neelakanta, "ATM telecommunications principles and implementation", CRC Press, Boca Raton, 2000.

[21] R. Onvural, "Asynchronous Transfer Mode Networks-performance Issues (2nd Ed.)", Arteck House Publishers, 1995.

[22] V. Paxson and S. Floyd, "Wide area traffic: the failure of Poisson modeling", IEEE/ACM Trans. on Networking 1995, 3(3), pp226-244.

[23] K. Peter, "Stochastic Linear Programming", Springer-Verlag, Berlin, 1976.

[24] S. Tanenbaum, "Computer Networks (3rd ed)", Prentice-Hall Inc, 1996.

[25] G. Tintner, "The pure theory of production under technological risk and uncertainty", Econometrica, 1941, 9, pp.298-304.